Quadratic Higher-Order Criteria for Iterative Blind Separation of a MIMO Convolutive Mixture of Sources

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***Abstract*— This work deals with the problem of source separation in the case when the observations result from a MIMO convolutive mixing system. In a blind framework, higher-order contrast functions have been proved to be efficient for extracting sources. Inspired by a semi-blind approach, we propose new contrast functions for blind signal separation which make use of reference signals. The main advantage of this approach consists in the quadratic form of these criteria: the extraction of one source hence reduces to a simple optimization task for which fast and efficient algorithms are available. The separation of the other sources from the mixture is then carried out by an iterative deflation method. Furthermore, these contrasts are shown to be valid for both i.i.d. and non i.i.d. source signals. The performance offered by these criteria is investigated through simulations: they appear as very appealing tools compared with some classical contrast functions.**

***Index Terms*— contrast functions, blind source separation, higher order statistics, cumulants, equalization**

I. Introduction

A. Generalities

The problem of blind equalization of Linear Time Invariant (LTI) systems appears in a wide variety of applications. Multi-user environments have brought the necessity to equalize the received signals both in space and time in order to reduce inter-symbol and co-channel interferences. In this context, the use of blind algorithms offers many advantages: the possibility to consider rapidly time-varying channels, no need for a training sequence and hence a potential increase of the available bandwidth. In many other fields such as array processing, passive sonar, seismic exploration, speech processing or biology, blind equalization has also proved to be useful. The corresponding generic problem is then generally referred to as the blind source separation (BSS) one. In this context, we propose separation criteria which allow to tackle the issue through an iterative separation method. We first give a short review of previous works on the subject.

Depending on the characteristics of the input signal(s) and on the number of inputs and outputs of the considered linear system, the blind equalization problem can be formulated in different ways. Historically, the first works considered a single sequence of independent and identically distributed (i.i.d.) random variables, corresponding to the input signal, and a single output signal. This problem has been considered in numerous works, using certain signal properties like higher-order statistics [40, 3, 21] or constant modulus [15, 47].

More recently, the research community has addressed more challenging models where the number of inputs is greater than one [50, 49, 43, 45]. This multichannel case has been a promising research area for the last few years, for which strong connections have been established with the instantaneous blind source separation problem. The latter one has become very popular and is often referred to as Independent Component Analysis (ICA). The underlying model assumes that there is no intersymbol interference but only co-channel interference (see e.g. [9, 24, 30, 39]). More complicated models have been studied (e.g. non linear [44], convolutive, ...). The problem of separating convolutive mixtures, which is addressed in this paper, has been tackled by different methods. On the one hand, it is possible in the case of non i.i.d. sources, to exploit either the non-stationnarity or the spectral diversity and to develop methods based on second-order statistics [38, 27, 26]. On the other hand, it is possible to use higher-order statistics and the corresponding methods may be classified as follows:

- Contrast-based approaches known in the case of instantaneous mixtures have been generalized to the convolutive case [10, 37]. More recently, contrasts have been exhibited in the case of non i.i.d. sources [31, 6]. Jacobi-like methods have been proposed for the maximization of such criteria [11]. These approaches, which extract all the sources simultaneously, often suffer from the need to perform a prewhitening stage and from the existence of spurious local maxima of the optimized criteria.
- It has long been noticed that convolutive mixtures can be translated into instantaneous ones at each frequency. This is the basic idea for several frequency domain methods [8, 13, 12, 29]. All these methods have to take into account a permutation and scaling ambiguity at each frequency. Furthermore, to provide good results, these methods may require a number of samples even larger than other methods.
- Iterative methods extract the sources one by one and therefore rely on the ability to extract one source (see e.g. [20, 48, 42]). Depending on the context, they have been referred to as “deflation” techniques [33, 48, 42], hierarchical [46] or multistage [19] methods. These iterative methods are very interesting because they remain valid in the case of non i.i.d. signals [42] and because the corresponding separation criteria may not present any spurious local maxima [42].

In this paper, we consider a Multi-Input / Multi-Output (MIMO) LTI system with possibly non i.i.d. input source signals, even though we mainly focus our attention on the i.i.d. case. Our approach relies on a Multi-Input / Single-output (MISO) inverse filter criteria based on higher-order statistics followed by a deflation procedure to extract all the source signals. We propose a new objective function which makes use of so-called reference signals [2, 4, 7] and we show that it is a contrast function allowing to extract one source signal. Hence, the source separation problem can be solved iteratively by successive global maximizations of this function. The proposed objective function has the great advantage of depending quadratically on the searched parameters. This leads to a simplified optimization scheme, thus significantly speeding up the source estimation process. We further investigate the conditions required for the reference signals and show that they are widely satisfied. This allows to use the proposed criteria in a blind context. The resulting optimization problems are also addressed.

It is worth noting that our approach is related to other works. First, we have to mention that other approaches have previously been called "reference"-based, although they are not directly linked to our approach and rely on second order statistics: so, in [28] a method is proposed for the separation of instantaneous mixtures. The method in [14] deals with the equalization of a single source mixed on several sensors and it has been generalized to convolutive MIMO mixtures in [34].

In the present paper, we call "reference" signal any signal fixed prior to the separation, whose cross-statistics with the observations allows to build separation criteria. The reference signal may either stem from a semi-blind approach, or may be chosen from the observations. The papers most directly linked to our approach are [35, 1] on the one hand and [21, 23, 22] on the other hand. The first ones both deal with instantaneous mixtures and require a prewhitening of the observation vector. The seminal work in [21, 23, 22] yields a method called EVA (Eigenvector Algorithm) that is applied to scalar deconvolution problems only (i.e. a single source is filtered by different channels and observed on several sensors). Moreover in the latter work, the source signals are assumed to be i.i.d. although some classes of signals of interest such as CPM (Continuous Phase Modulated) ones do not satisfy this assumption. The original contribution of our work is to address the general case of a convolutive mixture of several sources, which may be non i.i.d.

The model, notations and key assumptions are given in Section II. Section III proves the validity of the proposed criteria for source separation. In particular, in Section III-C, we give the conditions for these criteria to be used in a totally blind context. The case of non i.i.d. sources is studied in Section III-D. Section IV gives some detailed explanations about the optimization procedure and the possibility to extend the method to a MIMO separating algorithm. Finally, simulation results are given in Section V and Section VI concludes the paper.

B. Notations

In the whole paper, n stands for a generic integer ($n \in \mathbb{Z}$). Bold upper (resp. lower) case letters are used for matrices (resp. vectors). $\mathbf{M}[z] \triangleq \sum_{n\in\mathbb{Z}} \mathbf{M}(n) z^{-n}$ denotes the transfer function of the LTI system $\{\mathbf{M}\}$ with impulse response $\mathbf{M}(n)$. All quantities throughout the paper may be either real or complex-valued. $.^*$ stands for the complex conjugate, $.^T$ for matrix transpose, $.^H$ for matrix hermitian transpose. To provide a general formulation in Section III-B, we introduce the notation $y^{(*)_i}(n)$, where for $i \in \{1, 2\}$, the symbol $.^{(*)_i}$ means an optional complex conjugation, that is, for each index i, we have either $y^{(*)_i}(n) = y(n)$ or $y^{(*)_i}(n) = y^*(n)$. Finally $\mathrm{Cum}\{.\}$ stands for the the cumulant of any set of random variables, $\mathbf{I}$ is the identity matrix and δ_n stands for the Kronecker symbol, i.e. $\delta_n = 1$ if $n = 0$ and 0 otherwise.

II. Model and problem formulation

We consider an observed Q-dimensional ($Q \in \mathbb{N}, Q \geq 2$) discrete-time signal. Its nth sample is denoted by the column vector $\mathbf{x}(n)$. Assuming a noise-free model, the observation $\mathbf{x}(n)$ results from an LTI multichannel system described by the input-output relation:

$$\mathbf{x}(n) = \sum_{k\in\mathbb{Z}} \mathbf{M}(k)\mathbf{s}(n-k) \triangleq \{\mathbf{M}\}\mathbf{s}(n). \qquad (1)$$

In the above equation, $\mathbf{M}(n)$ represents a (Q, N) matrix corresponding to the impulse response of the LTI mixing system, which is denoted by $\{\mathbf{M}\}$ for simplicity. The vector of source signals (or *sources*) $\mathbf{s}(n)$ is an N-dimensional ($N \in \mathbb{N}^*$) unknown and unobserved column vector. The objective is to restore the sources *blindly*, that is from the only use of the observations. For this to be achievable, the following assumptions must be made on the mixing system:

- A0. The LTI mixing system is *stable* (i.e. for all (i, j), the (i, j)th element $M_{ij}(n)$ of the matrix $\mathbf{M}(n)$ forms an absolutely summable sequence: $\sum_{n\in\mathbb{Z}} |M_{ij}(n)| < \infty$, $\forall(i, j)$).
- A0'. The LTI mixing system is left *invertible*, that is there exists a stable LTI system $\{\mathbf{W}\}$ with impulse response $\mathbf{W}(n)$ such that the global LTI system $\{\mathbf{G}\} \triangleq \{\mathbf{W} \star \mathbf{M}\}$ with impulse response $\mathbf{G}(n) \triangleq \sum_{k\in\mathbb{Z}} \mathbf{W}(k)\mathbf{M}(n-k)$ corresponds to an identity LTI multichannel system, i.e. $\mathbf{G}(n) = \mathbf{I}\delta_n$.

One should notice that A0' can be satisfied only if there are more observed signals than sources ($Q \geq N$). More precisely, A0' is satisfied if and only if for all ω, the matrix $\mathbf{M}[e^{\imath\omega}]$ has full rank N (see e.g. [17]). In the case of MIMO-FIR (Finite Impulse Response) systems, one can give a more precise result [18]: a left MIMO-FIR inverse system exists if and only if $\mathbf{M}[z]$ has full column rank for all $z \in \mathbb{C}, z \neq 0$.

Our approach being an iterative one, we will focus on the extraction of a single source (the extraction of all sources is done through a deflation procedure which will be described in Section IV-C). In this MISO context, the considered problem consists in estimating one row of $\{\mathbf{W}\}$, that is a $(1, Q)$ LTI vector filter $\{\mathbf{w}\}$, called an equalizer, and with impulse response $\mathbf{w}(n)$, such that the scalar signal

$$y(n) = \sum_{k\in\mathbb{Z}} \mathbf{w}(k)\mathbf{x}(n-k) \tag{2}$$

restores one of the components $s_i(n), i \in \{1,\ldots,N\}$, of the source vector. In order to obtain tractable expressions, we define the corresponding $(1,N)$ global LTI vector filter $\{\mathbf{g}\}$ by its impulse response $\mathbf{g}(n) \triangleq \sum_{k\in\mathbb{Z}} \mathbf{w}(k)\mathbf{M}(n-k)$, and then we have

$$y(n) = \sum_{k\in\mathbb{Z}} \mathbf{g}(n-k)\mathbf{s}(k) \triangleq \{\mathbf{g}\}\mathbf{s}(n) \ . \tag{3}$$

To be able to carry out the estimation blindly, assumptions about the source signals are required. In this paper, we will adopt the following assumptions:

A1. The source vector components $s_i(n), i \in \{1,\ldots,N\}$ are stationary and zero-mean random processes with unit variance. They are also *statistically mutually independent* (typically up to the order of the considered cumulants). The source signal correlation sequences are definite positive[1] and are respectively denoted by $\gamma_i(k) \triangleq \mathrm{E}\{s_i(n)s_i^*(n-k)\}$ where $k \in \mathbb{Z}$ and $i \in \{1,\ldots,N\}$.

Now it has to be noticed that there exist some inherent undetermined factors in the estimation of the source signals. Indeed, they can be recovered "only" up to a permutation and to a scalar filtering. For this reason, the equalization of one source signal is said to be achieved when there exists an index $i_0 \in \{1,\ldots,N\}$ and a non-zero scalar filter with impulse response $g(n)$, such that the ith filter component in $\mathbf{g}(n)$ reads

$$g_i(n) \triangleq (\mathbf{g}(n))_i = \alpha g(n)\delta_{i-i_0} \ , \tag{4}$$

where $\alpha \in \mathbb{C}, \alpha \neq 0$. The above relation is called the "*equalization condition*" and expresses the fact that $y(n)$ is equal to one source signal, $s_{i_0}(n)$, up to a scalar filtering.

Notice that the above equalization condition can be made more restrictive when all source signals are also assumed to be sequences of independent and identically distributed (i.i.d.) complex random variables. Indeed, in such a case, it is classically said that the equalization is realized when the scalar filter $g(n)$ reduces to a simple delay. This reads:

$$\exists l \in \mathbb{Z} \quad g_i(n) \triangleq (\mathbf{g}(n))_i = \alpha\delta_{n-l}\delta_{i-i_0} \ . \tag{5}$$

For convenience, we introduce for any index $j \in \{1,...,N\}$ and any scalar filter with impulse response $h(n)$ its j-norm:

$$\|h\|_j^2 \triangleq \sum_{(k_1,k_2)\in\mathbb{Z}^2} h(k_1)h^*(k_2)\gamma_j(k_2-k_1). \tag{6}$$

One can easily see that $\|h\|_j^2$ is the variance of the signal obtained when filtering the source $s_j(n)$ by the filter with impulse response $h(n)$. Since the sources have unit variance, one can restrict the multiplicative factor in (4) and (5) to $|\alpha| = 1$ by imposing the constraint $\mathrm{E}\{|y(n)|^2\} = 1$. Defining the (weighted) ℓ^2-norm of the global $(1,N)$ filter $\{\mathbf{g}\}$ by

$$\|\mathbf{g}\|^2 \triangleq \sum_{i=1}^{N} \|g_i\|_i^2 \tag{7}$$

[1] A summable correlation sequence is definite positive when the corresponding spectrum density is positive.

one can see that the constraint $\mathrm{E}\{|y(n)|^2\} = 1$ is then equivalent to $\|\mathbf{g}\|^2 = 1$, i.e. to having a unit-norm global filter. Note that for i.i.d. source signals, this condition simply becomes:

$$\sum_{i=1}^{N}\sum_{k\in\mathbb{Z}} |g_i(k)|^2 = 1. \tag{8}$$

Let us finally introduce some useful notations. In the following, we denote by $\mathcal{S}$ the set of source signals satisfying assumptions A1. We also denote by $\mathcal{G}_1$ the set of unit norm vector filters and by $\mathcal{G}_{1e}^{i_0}$ the subset of filters in $\mathcal{G}_1$ satisfying the equalization condition (4). Note that, when $\{\mathbf{g}\} \in \mathcal{G}_{1e}^{i_0}$, in addition to the constraint $|\alpha| = 1$, we may suppose in (4) that $\|g\|_{i_0} = 1$. We denote by $\mathcal{G}_{1ed}^{i_0}$ the subset of $\mathcal{G}_{1e}^{i_0}$ when $g(n) = \delta_{n-l}$ (i.e. the set of global filters satisfying the equalization condition (5)). Finally, we denote $\mathcal{Y}$ the set of the output MISO equalizer as defined in (3), when the source signal belongs to $\mathcal{S}$ and the global system belongs to $\mathcal{G}_1$.

III. A GENERAL FAMILY OF CONTRAST FUNCTION

A. Contrast function

Before presenting our results, we recall some useful definitions in the context of MISO equalization. It is well-known that one of the most interesting approaches to the blind equalization problem consists in the use of an appropriate contrast function. Basically, a contrast function plays the role of an objective function in the sense that its (global) maximization allows us to solve the problem. In this way, the source separation problem becomes an optimization one. Moreover, identifiability conditions are provided by the definition domain of the considered contrast function. In accordance with the considered method, which is iterative and relies on successive MISO extractions of each of the sources, we introduce the following definition of a contrast function for i.i.d. source signals.

Definition 1 (contrast function for ***i.i.d. sources****):* Let $\mathsf{C}\{\cdot\}$ be a function from $\mathcal{Y}$ to $\mathbb{R}$. It is called a contrast function or simply a contrast when it satisfies the two following properties:

p1. There exists $(i_0,k_0) \in \{1,\ldots,N\}\times\mathbb{Z}$ such that

$$\forall y(n) \in \mathcal{Y} \quad \mathsf{C}\{y(n)\} \leq \mathsf{C}\{s_{i_0}(n-k_0)\}. \tag{9}$$

p2. The equality holds in (9) for any $(i_0,k_0) \in \{1,\ldots,N\}\times\mathbb{Z}$ if and only if the global filter is separating, i.e. $\{\mathbf{g}\} \in \mathcal{G}_{1ed}^{i_0}$.

According to the above definition, the global maximization of a contrast function leads to the extraction of one source up to a delay and a multiplicative factor.

However, this definition cannot be used for non i.i.d. source signals whose extraction based on mutual independence criteria can only be guaranteed up to a scalar filtering. This is the reason why a weaker definition of a contrast is now given, which will be used for non i.i.d. source signals.

Definition 2 (contrast function for ***non i.i.d. sources****):* Let $\mathsf{C}\{\cdot\}$ be a function from $\mathcal{Y}$ to $\mathbb{R}$. It is called a contrast function when it satisfies the two following properties:

p1'. For all possible output of the equalizer:

$$\mathsf{C}\{y(n)\} \leq \max_{i=1}^{N} \sup_{\mathbf{g}\in\mathcal{G}_{1e}^{i}} \mathsf{C}\{\{\mathbf{g}\}\mathbf{s}(n)\}. \quad (10)$$

p2'. The equality holds in (10), if and only if there exists i_0 such that $\mathbf{g} \in \mathcal{G}_{1e}^{i_0}$.

It is clear that if a function is a contrast in the sense of Definition 1 then it is also a contrast in the sense of Definition 2. The converse property is not true in general since the separation as expressed in Section II constitutes a weaker property for non i.i.d. sources than for i.i.d. sources.

B. Case of i.i.d. source signals

To provide a better insight in the results, we first consider the relatively simple case of i.i.d. source signals. The proofs will be extended to the non i.i.d. case in Section III-D. One of the contributions of the paper consists in proposing criteria based on R-th order ($R \geq 3$) cross-cumulants, where $R-2$ variables are fixed. This choice yields a quadratic dependence with respect to the optimized parameters which greatly simplifies the optimization task.

According to the notations in Section I-B, we define the following R-th order ($R \geq 3$) cumulant (where the optional complex conjugate $.^{(*)_1}$ and $.^{(*)_2}$ are not necessarily equal):

$$\kappa_{R,\mathbf{z}}\{y(n)\} = \mathrm{Cum}\{y^{(*)_1}(n), y^{(*)_2}(n), z_1(n), \ldots, z_{R-2}(n)\} \quad (11)$$

where $z_i(n), i \in \{1, \ldots, R-2\}$ are signals fixed prior to the separation. In our previous works [4, 2], these signals have been referred to as *reference* signals which have been determined from prior information, but we will prove later (see Section III-C) that they may be chosen in a rather arbitrary way. We now define the following function:

$$\mathcal{C}_{R,\mathbf{z}}\{y(n)\} \triangleq |\kappa_{R,\mathbf{z}}\{y(n)\}| \ , \qquad R \geq 3. \quad (12)$$

We will prove that $\mathcal{C}_{R,\mathbf{z}}\{y(n)\}$ is a contrast function. To this end, we define the following supremum,

$$\kappa_R^{\max} = \max_{j=1}^{N} \sup_{k\in\mathbb{Z}} |\kappa_{R,\mathbf{z}}\{s_j(n-k)\}|. \quad (13)$$

We moreover assume that it is reached and hence bounded:

A2. There exists (j_0, k_0) such that:

$$\kappa_R^{\max} = |\kappa_{R,\mathbf{z}}\{s_{j_0}(n-k_0)\}| < +\infty. \quad (14)$$

Under the above assumption, which will be proved to be widely satisfied (see Proposition 2), we are able to state the following proposition:

Proposition 1: In the case of i.i.d. source signals, under Assumption A2., the function $\mathcal{C}_{R,\mathbf{z}}$ is a contrast over the set of unit norm row filters ($\|\mathbf{g}\| = 1$) if and only if the set

$$\mathcal{I} \triangleq \{(j,k) \in \{1, \ldots, N\} \times \mathbb{Z} \mid |\kappa_{R,\mathbf{z}}\{s_j(n-k)\}| = \kappa_R^{\max}\} \quad (15)$$

contains a single element.

Proof: The proof is given in Appendix I ■

C. Blind use of the proposed criteria

1) Reference signal: We have not specified so far how to choose the signals $z_i(n)$. The simplest way to make this choice consists in assuming that each signal is obtained by a MISO filtering of the sources:

$$\forall i \in \{1, \ldots, R-2\} \quad z_i(n) = \sum_{k\in\mathbb{Z}} \mathbf{t}^{(i)}(n-k)\mathbf{s}(k) \ . \quad (16)$$

where for all i, $\mathbf{t}^{(i)}(n)$ is the impulse response of a $(1, Q)$ stable vector filter. In a semi-blind context, $z_i(n), i \in \{1, \ldots, R-2\}$ can be called *reference signals* and $\mathbf{t}^{(i)}(n), i \in \{1, \ldots, R-2\}$ can be viewed as the impulse responses of *reference systems*. In choosing the reference signals as filtered versions of the sources, one can easily ensure that Assumption A2. is fullfilled, as stated by the following Proposition.

Proposition 2: If the filters with impulse responses $\mathbf{t}^{(i)}(n), i \in \{1, \ldots, R-2\}$ are stable, then assumption A2. is satisfied.

Proof: Similarly to the proof of Proposition 1, in order to simplify notations, we restrict the study to the case when $y^{(*)_1}(n) = y^{(*)_2}(n) = y(n)$. Using cumulant multilinearity and source independence, one obtains (for any given j in $\{1, \ldots, N\}$):

$$|\kappa_{R,\mathbf{z}}\{s_j(n-k)\}| = |\mathrm{C}_R\{s_j(n)\}| \ | \prod_{i=1}^{R-2} t_j^{(i)}(k)| \quad (17)$$

where $t_j^{(i)}(k)$ is the j-th component of $\mathbf{t}^{(i)}(k)$ and where we have defined the R-th order auto-cumulant $\mathrm{C}_R\{s_j(n)\} \triangleq \mathrm{Cum}\{s_j(n), s_j(n), s_j(n), \ldots, s_j(n)\}$. Because of the stability of the filters, the sequences $(t_j^{(i)}(k))_{k\in\mathbb{Z}}$ are summable and so is the sequence $\left(\prod_{i=1}^{R-2} t_j^{(i)}(k)\right)_{k\in\mathbb{Z}}$. This implies that the latter sequence converges to 0 when $|k| \to \infty$ and, hence, the supremum over $k \in \mathbb{Z}$ is reached in (13), showing that Assumption A2. holds. ■

2) Randomly driven reference system: It seems natural to wonder whether the validity condition for the proposed contrast as given in Proposition 1 is restrictive or not. Actually, the following proposition shows that this condition is weak, since it appears that $z_i(n), i \in \{1, \ldots, R-2\}$ can be chosen almost arbitrarily, unlike what has been done in [4]:

Proposition 3: Assume that we have $\kappa_R^{\max} > 0$ in Equation (14). If the filters $\{\mathbf{t}^{(i)}\}$, $1 \leq i \leq R-2$ have finite impulse response (FIR) and if their coefficients have been chosen randomly distributed according to a continuous joint probability density function (supported on a set of non-zero measure), then almost surely the set $\mathcal{I}$ has one single element and $\mathcal{C}_{R,\mathbf{z}}$ is a contrast.

Proof: Denote by (j_0, k_0) a pair of indices satisfying Equation (14) (such a pair exists according to Proposition 2 and the FIR assumption). From Equation (17), one can see that for any $(j,k) \neq (j_0, k_0)$, we have $(j,k) \in \mathcal{I}$ if and only if

$$|\mathrm{C}_R\{s_j(n)\} \prod_{i=1}^{R-2} t_j^{(i)}(k)| = |\mathrm{C}_R\{s_{j_0}(n)\} \prod_{i=1}^{R-2} t_{j_0}^{(i)}(k_0)| \ , \quad (18)$$

which is almost surely false if the coefficients are driven from a continuous joint probability density function. ■

One should notice that the above proposition still holds true when all reference filters are identical ($\forall i \in \{1,\ldots,R-2\}, \mathbf{t}^{(i)} = \mathbf{t}$). This case of particular interest will often be considered in the following.

Finally, note also that all the arguments used to prove Propositions 1, 2 and 3 still apply if some of the "reference signals" $z_i(n), i \in \{1,\ldots,R-2\}$ are replaced by their complex conjugates. This means that the expression in Equation (11) can be generalized by conjugating or not each term in the expression of the cumulant. For readability, we did not introduce additional notations on the reference signals to make it explicit.

3) Example of 4th or 3rd order cumulants and comments: We here address the particular case when 4-th order cumulants are considered with an equal number of conjugated and non conjugated terms and when one reference signal is used only. More precisely, we have $R = 4$, $z_1(n) = z_2^*(n) \triangleq z(n)$ and the contrast reads:

$$\begin{aligned}&\mathcal{C}_{4,z}\{y(n)\} \triangleq |\mathrm{Cum}\{y(n), y^*(n), z(n), z^*(n)\}|\\ &\text{with:} \quad z(n) = \sum_{k\in\mathbb{Z}} \mathbf{t}(n-k)\mathbf{s}(k)\ . \end{aligned} \tag{19}$$

Notice that if the reference signal $z(n)$ is replaced by $y(n)$, the function $\mathcal{C}_{4,z}\{y(n)\}$ was already shown to be a contrast in [48] for the case of i.i.d. sources and in [42] for non i.i.d. sources. At this point, one may wonder why no assumption has been made on the 4-th order cumulants of the sources, which must usually be non zero [48, 42]. In fact, this assumption is taken into account in Proposition 1. Indeed, in order to satisfy the assumption of Proposition 1, we must have $\kappa_R^{\max} > 0$ which necessarily implies that at least one source should have a non zero 4-th order auto-cumulant. This justifies that the following assumption is required by our method:

A3. At least one of the sources has non zero 4-th order auto-cumulant, i.e. there exists $i \in \{1,\ldots,N\}$,

$$C_4\{s_i(n)\} \triangleq \mathrm{Cum}\{s_i(n), s_i^*(n), s_i(n), s_i^*(n)\} \neq 0.$$

More precisely, one can remark from the proof of Proposition 1 that the maximization of the contrast $\mathcal{C}_{4,z}\{y(n)\}$ leads to the extraction of a source such that there exists (j_0, k_0) satisfying Equation (14), that is $|\kappa_{4,z}\{s_{j_0}(n-k_0)\}|$ reaches the maximum value. By writing $|\kappa_{4,z}\{s_{j_0}(n-k_0)\}| = |t_{j_0}(k_0)|^2 |\mathrm{Cum}\{s_{j_0}(n), s_{j_0}^*(n), s_{j_0}(n), s_{j_0}^*(n)\}|$ one sees that the maximum value of $|\kappa_{4,z}\{s_{j_0}(n-l_0)\}|$ results from a trade-off, where either the auto-cumulant or the coefficient $|t_{j_0}(k_0)|$ takes a large value. The latter condition can be interpreted as the prominence of the j_0-th source with a delay k_0 in the reference signal. Finally, if some of the sources in the mixture have vanishing auto-cumulants, one should remark that it still possible to extract the sources with non zero auto-cumulants.

A similar discussion holds, under minor modifications, for cumulants of any order. In particular, 3-rd order cumulants will be used in the simulations. In this case, the contrast reads $\mathcal{C}_{3,z}\{y(n)\} \triangleq |\mathrm{Cum}\{y(n), y(n), z(n)\}|$ and assumption A3. becomes: there exists $i \in \{1,\ldots,N\}$ such that $\mathrm{Cum}\{s_i(n), s_i(n), s_i(n)\} \neq 0$. One should pay attention to the fact that this assumption is not satisfied by sources having a symmetric probability density function. Odd-order cumulants cannot be used in the latter case.

D. Case of non i.i.d. source signals

1) Validity of the proposed contrast function for non i.i.d. sources: Now, inspired from [42], we show how the results can be adapted to the case of non i.i.d. source signals. Similarly to Equation (13), we define:

$$\begin{aligned}&\forall i \in \{1,\ldots,N\}, \quad \mathcal{M}_i \triangleq \sup_{\{\mathbf{g}\}\in\mathcal{G}_{1e}^i} \mathcal{C}_{R,\mathbf{z}}\{\{\mathbf{g}\}\mathbf{s}(n)\}\\ &\text{and} \qquad \mathcal{M}_{\max} \triangleq \max_{i=1}^{N} \mathcal{M}_i. \end{aligned} \tag{20}$$

The following set will play a role identical to the one in Equation (15):

$$\mathcal{I}' \triangleq \{j \in \{1,\ldots,N\} \mid \mathcal{M}_j = \mathcal{M}_{\max}\}. \tag{21}$$

Finally, Assumption A2. should be modified as follows:

A4. For all $i \in \mathcal{I}'$, the supremum $\mathcal{M}_i$ in (20) is reached by a filter which is denoted by $\{\mathbf{g}_i^\sharp\} \in \mathcal{G}_{1e}^i$.

We are then able to state the following result:

Proposition 4: In the case of non i.i.d. signals, under assumption A4., the function $\mathcal{C}_{R,\mathbf{z}}$ is a contrast over the set of unit norm filters ($\|\mathbf{g}\| = 1$) if and only if the set $\mathcal{I}'$ contains a single element.

Proof: The proof is given in Appendix II. ■

2) A sufficient condition for the validity of Proposition 4: In the particular case when only one reference signal given by $z(n) = \sum_{k\in\mathbb{Z}} \mathbf{t}(n-k)\mathbf{s}(k)$ is considered, the following Proposition gives a sufficient condition for Proposition 4 to hold true. Let us define for any $j \in \{1,\ldots,N\}$ the following R-th order auto-cumulant at time-lag $(p_1,\ldots,p_{R-1})$:

$$\begin{aligned}&\mathrm{C}_R^{(p_1,\ldots,p_{R-1})}\{s_j(n)\} \triangleq\\ &\qquad \mathrm{Cum}\{s_j(n), s_j(n+p_1),\ldots,s_j(n+p_{R-1})\}. \end{aligned} \tag{22}$$

For any $j \in \{1,\ldots,N\}$ we set:

$$\begin{aligned}&\|t_j\|_{\ell_1} \triangleq \sum_{k\in\mathbb{Z}} |t_j(k)| \qquad \|t_j\|_{\ell_\infty} \triangleq \sup_{k\in\mathbb{Z}} |t_j(k)|\\ &\text{and:} \quad \|\mathrm{C}_R^{(\cdot)}\{s_j(n)\}\|_{\ell_1} \triangleq \sum_{p_1,\ldots,p_{R-1}} |\mathrm{C}_R^{(p_1,\ldots,p_R)}\{s_j(n)\}|. \end{aligned}$$

We further assume that the correlation sequences of the sources are summable and, as they have been assumed positive definite, the j-th source has a continuous spectrum density which is lower-bounded by a constant $\Gamma_j^{\min} > 0$.

Proposition 5: A sufficient condition for Proposition 4 to hold true, is that there exists $j_0 \in \{1,\ldots,N\}$ such that, for all $j \neq j_0$,

$$\begin{aligned}&\|t_{j_0}\|_{\ell_\infty}^{R-2} |\mathrm{C}_R^{(0,\ldots,0)}\{s_{j_0}(n)\}| >\\ &(\|t_{j_0}\|_{\ell_1}^{R-2} - \|t_{j_0}\|_{\ell_\infty}^{R-2}) \sup_{(l_1,\ldots,l_2)\neq(0,\ldots,0)} |\mathrm{C}_R^{(0,l_1,\ldots,l_{R-2})}\{s_{j_0}(n)\}|\\ &\qquad + (\Gamma_j^{\min})^{-1} \|t_j\|_{\ell_\infty}^{R-2} \|\mathrm{C}_R^{(\cdot)}\{s_j(n)\}\|_{\ell_1}. \end{aligned} \tag{23}$$

Proof: The proof is given in Appendix III. ■

One notices that the difference between the left-hand side and the right-hand side of the above inequality is all the more important as $|\mathrm{C}_R^{(0,\ldots,0)}\{s_{j_0}(n)\}|$ is large or $\|t_{j_0}\|_{\ell_\infty}$ is close to $\|t_{j_0}\|_{\ell_1}$. This confirms our intuition that the extraction of a source should be easier when its kurtosis has a high value and when the reference is not too far from a separating filter extracting this source. Finally, in the case of i.i.d. sources, using the facts that $\mathrm{C}_R^{(p_1,\ldots,p_{R-1})}\{s_j(n)\}=0$ for $(p_1,\ldots,p_{R-1})\neq(0,\ldots,0)$ and $\Gamma_j^{\min}=1$, one can see that (23) reduces to:

$$\|t_{j_0}\|_{\ell_\infty}^{R-2}|\mathrm{C}_R^{(0,\ldots,0)}\{s_{j_0}(n)\}|>\|t_j\|_{\ell_\infty}^{R-2}|\mathrm{C}_R^{(0,\ldots,0)}\{s_j(n)\}|$$

which is exactly the validity condition formerly stated for i.i.d. sources. This illustrates that Propositions 4 and 5 generalize our previous results to non i.i.d. sources.

IV. Practical contrast optimization

We now derive an efficient method for the optimization of the previous contrasts. Moreover, we will describe an iterative method where the reference system is updated after each iteration by the output signal computed in previous iteration. This method which we refer to as a "fixed-point" method, exhibits improved performance in simulations.

A. Optimization method

We now consider the optimization of the proposed contrast through its practical implementation with FIR filters. Hence, in order to ensure Assumption A0', it is assumed that:

A5. The mixing system is an FIR filter with impulse response of length L. In addition, there are more sensors than sources ($Q\geq N$). Finally, the polynomial matrix z-transform $\mathbf{M}[z]$ of the filter $\{\mathbf{M}\}$ is irreducible.

Note by the way that irreducibility is widely satisfied as soon as there are strictly more sensors than sources ($Q>N$, see e.g. [16]). Assumption A5. ensures that the mixing filter admits a MIMO-FIR left inverse filter (see [16] or [25]), which can be assumed to be causal because of the delay indetermination and whose length is denoted by D.

The row vectors defining the impulse response of the MISO equalizer can always be stacked in the following $(1,QD)$ row vector:

$$\underline{\mathbf{w}}\triangleq\begin{pmatrix}\mathbf{w}(0) & \mathbf{w}(1) & \ldots & \mathbf{w}(D-1))\end{pmatrix} \tag{24}$$

We similarly define the $(QD,1)$ column vector

$$\underline{\mathbf{x}}(n)\triangleq\begin{pmatrix}\mathbf{x}(n)^T & \mathbf{x}(n-1)^T & \ldots & \mathbf{x}(n-D+1)^T\end{pmatrix}^T.$$

Using these notations it is straightforward to see that Equation (2) can be written as

$$y(n)=\underline{\mathbf{w}}\,\underline{\mathbf{x}}(n). \tag{25}$$

Hence the power of the output of the MISO equalizer reads

$$\mathrm{E}\{|y(n)|^2\}=\underline{\mathbf{w}}\mathbf{R}_{\underline{\mathbf{x}}}\underline{\mathbf{w}}^H \tag{26}$$

where $\mathbf{R}_{\underline{\mathbf{x}}}\triangleq\mathrm{E}\{\underline{\mathbf{x}}(n)\underline{\mathbf{x}}(n)^H\}$ is the covariance matrix of $\underline{\mathbf{x}}(n)$.

Let us now consider the cumulants in (11). Using the multilinearity property of cumulants, one can easily obtain

$$\kappa_{R,\mathbf{z}}\{y(n)\}=\underline{\mathbf{w}}^{(*)_1}\mathbf{C}_{\underline{\mathbf{x}},\mathbf{z}}\left(\underline{\mathbf{w}}^{(*)_2}\right)^T \tag{27}$$

where $\underline{\mathbf{w}}^{(*)_1}$ (resp. $\underline{\mathbf{w}}^{(*)_2}$) stands for the vector $\underline{\mathbf{w}}$ conjugated in the same way as $y^{(*)_1}(n)$ (resp. $y^{(*)_2}(n)$) in (11). The matrix $\mathbf{C}_{\underline{\mathbf{x}},\mathbf{z}}$ is defined component-wise as

$$\left(\mathbf{C}_{\underline{\mathbf{x}},\mathbf{z}}\right)_{i,j}=\mathrm{Cum}\{\underline{x}_i^{(*)_1}(n),\underline{x}_j^{(*)_2}(n),z_1(n),\ldots,z_{R-2}(n)\}$$

where $\underline{x}_i^{(*)_1}(n)$ (resp. $\underline{x}_j^{(*)_2}(n)$) are the i-th (resp. j-th) component of vector $\underline{\mathbf{x}}(n)$ conjugated as explained above.

Hence optimizing the contrast $\mathcal{C}_{R,\mathbf{z}}$ under the constraint of a unit norm MISO filter is equivalent to determine

$$\max|\underline{\mathbf{w}}^{(*)_1}\mathbf{C}_{\underline{\mathbf{x}},\mathbf{z}}\left(\underline{\mathbf{w}}^{(*)_2}\right)^T| \quad \text{under the constraint} \quad \underline{\mathbf{w}}\mathbf{R}_{\underline{\mathbf{x}}}\underline{\mathbf{w}}^H=1\,. \tag{28}$$

It can be noticed that the covariance matrix $\mathbf{R}_{\underline{\mathbf{x}}}$ is positive semidefinite and thus some of its eigenvalues can be zero. In such a case, the solution to the above maximization problem is not unique since any vector belonging to the nullspace $\ker\mathbf{R}_{\underline{\mathbf{x}}}$ of $\mathbf{R}_{\underline{\mathbf{x}}}$ can be added to the solution without changing the separator output (if $\underline{\mathbf{w}}^H\in\ker\mathbf{R}_{\underline{\mathbf{x}}}$, then $\underline{\mathbf{w}}\,\underline{\mathbf{x}}(n)$ obviously vanishes identically). Hence we can impose the following additional constraint:

$$\underline{\mathbf{w}}^H\in(\ker\mathbf{R}_{\underline{\mathbf{x}}})^\perp. \tag{29}$$

This constraint is introduced in order to ensure that a *unique* solution to (28) exists. This in turn allows to simplify further the problem (see Equation (36)) although $\mathbf{R}_{\underline{\mathbf{x}}}$ is not necessarily positive definite. Imposing (29) can be easily done by a projection onto the signal subspace spanned by the eigenvectors associated to all non zero eigenvalues. More precisely, as $\mathbf{R}_{\underline{\mathbf{x}}}$ is a covariance matrix, it can be decomposed as

$$\mathbf{R}_{\underline{\mathbf{x}}}=\mathbf{U}\mathbf{D}\mathbf{U}^H \tag{30}$$

where $\mathbf{D}$ is the square diagonal matrix corresponding to all the non zero eigenvalues and $\mathbf{U}$ is a matrix whose columns correspond to the family of orthonormal eigenvectors associated to these non zero eigenvalues. Let us now introduce the two following matrices

$$\mathbf{P}\triangleq\mathbf{U}\mathbf{D}^{\frac{1}{2}}\qquad\mathbf{Q}\triangleq\mathbf{D}^{-\frac{1}{2}}\mathbf{U}^H. \tag{31}$$

The constraint in (29) is then taken into account by setting

$$\underline{\mathbf{w}}=\underline{\tilde{\mathbf{w}}}\mathbf{Q} \tag{32}$$

which leads to

$$\underline{\tilde{\mathbf{w}}}\triangleq\underline{\mathbf{w}}\mathbf{P}. \tag{33}$$

Then

$$\underline{\mathbf{w}}\mathbf{R}_{\underline{\mathbf{x}}}\underline{\mathbf{w}}^H=\underline{\mathbf{w}}\mathbf{P}\mathbf{P}^H\underline{\mathbf{w}}^H=\underline{\tilde{\mathbf{w}}}\underline{\tilde{\mathbf{w}}}^H \tag{34}$$

and

$$\kappa_{R,\mathbf{z}}\{y(n)\}=\underline{\mathbf{w}}^{(*)_1}\mathbf{C}_{\underline{\mathbf{x}},\mathbf{z}}\left(\underline{\mathbf{w}}^{(*)_2}\right)^T=\underline{\tilde{\mathbf{w}}}^{(*)_1}\tilde{\mathbf{C}}_{\underline{\mathbf{x}},\mathbf{z}}\left(\underline{\tilde{\mathbf{w}}}^{(*)_2}\right)^T$$

where

$$\tilde{\mathbf{C}}_{\underline{\mathbf{x}},\mathbf{z}}\triangleq\mathbf{Q}^{(*)_1}\mathbf{C}_{\underline{\mathbf{x}},\mathbf{z}}\left(\mathbf{Q}^{(*)_2}\right)^T. \tag{35}$$

Finally the maximization problem in (28) takes the more classical form:

$$\max|\underline{\tilde{\mathbf{w}}}^{(*)_1}\tilde{\mathbf{C}}_{\underline{\mathbf{x}},\mathbf{z}}\left(\underline{\tilde{\mathbf{w}}}^{(*)_2}\right)^T| \quad \text{under the constraint} \quad \underline{\tilde{\mathbf{w}}}\,\underline{\tilde{\mathbf{w}}}^H=1\,. \tag{36}$$

Note that the transformation on $\underline{\mathbf{w}}$ given by (33) corresponds to a prewhitening step. Then, the above optimization problem can be readily solved in the following two particular cases:

- if all quantities are real-valued, in which case the complex conjugation has no effect and the matrix $\tilde{\mathbf{C}}_{\underline{\mathbf{x}},\mathbf{z}}$ is real symmetric;
- if in Equation (11) which defines the contrast, each signal appears twice, once with a complex conjugate and once without. In such a case, the matrix $\tilde{\mathbf{C}}_{\underline{\mathbf{x}},\mathbf{z}}$ is hermitian symmetric and (36) amounts to the maximization of $|\tilde{\underline{\mathbf{w}}}\tilde{\mathbf{C}}_{\underline{\mathbf{x}},\mathbf{z}}\tilde{\underline{\mathbf{w}}}^H|$ under the constraint $\tilde{\underline{\mathbf{w}}}\,\tilde{\underline{\mathbf{w}}}^H = 1$.

In both cases, the solution is given by the eigenvector of $\tilde{\mathbf{C}}_{\underline{\mathbf{x}},\mathbf{z}}$ associated to the eigenvalue with largest modulus. In practice it can be obtained by an Singular Value Decomposition (SVD) of the considered matrix.

The advantages of the approach are twofold: first, the global maximum of the contrast can be reached and the optimization task is not made difficult by the possible existence of spurious local maxima. Second, the optimization does not require any iterative gradient-like algorithm which appears to be time consuming because of their slow convergence and the requirement to perform numerous contrast/gradient estimations. It follows that the optimization time is very significantly reduced since the estimation step is performed only once.

B. A "fixed-point" like method

We assume, in this paragraph, that only one reference system is used. This means that for all $i \in \{1, \ldots, R-2\}$, we have $z_i(n) = z(n), \forall n \in \mathbb{Z}$ (or possibly $z_i(n) = z^*(n)$ in the complex case). For clarity, we will write $\kappa_{R,z}$ and $\mathcal{C}_{R,z}$ instead of the previously used notations $\kappa_{R,\mathbf{z}}$ and $\mathcal{C}_{R,\mathbf{z}}$.

According to Proposition 3, the criterion $\mathcal{C}_{R,z}$ in (12) where the reference signals $z_i(n), i \in \{1, \ldots, R-2\}$ have all been chosen equal $z_i(n) = z(n)$, $\forall i$, constitutes almost surely a contrast when $z(n)$ is the output of a $(1, Q)$ FIR randomly driven row filter operating on the observed signals.

However, following the discussion in Section III-C.3, one could expect that the better the reference system separates the j_0-th source signal, the better the result of the extraction of the j_0-th source signal should be, in the presence of estimation errors. Furthermore, the output obtained after maximization of $\mathcal{C}_{R,z}$ should be closer to the extracted source than the reference one. Based on this observation, an iterative procedure is proposed. Basically, the output which has been previously obtained by the maximization of $\mathcal{C}_{R,z}$ serves as a "new" (or updated) reference signal. More precisely, using the same notations as in Section IV-A, the iterative procedure consists of the following steps:

1) choose or draw randomly a vector $\underline{\mathbf{w}}^0$
2) for $\ell \in \{1, 2, \ldots, \ell_{\max}\}$ do:
   - fix the reference to $z(n) = \underline{\mathbf{w}}^{\ell-1}\mathbf{x}(n)$,
   - $\underline{\mathbf{w}}^\ell = \arg\max_{\mathbf{w}} \mathcal{C}_{R,z}\{y(n)\}$ where $y(n) = \underline{\mathbf{w}}\underline{\mathbf{x}}(n)$.
3) The separating filter is given by the coefficients $\underline{\mathbf{w}}^{\ell_{\max}}$.

One can see that $\underline{\mathbf{w}}^\ell$ is obtained from $\underline{\mathbf{w}}^{\ell-1}$ from the function given by the $\arg\max$ expression. This is the reason why this algorithm has been called "fixed-point" like. Notice that a similar method was proposed for SISO deconvolution problems in [21]. We have observed in simulations that a few iterations $\ell_{\max}$ of the above method allows to improve the performance obtained with a single maximization of the contrast function using a randomly driven reference system. In particular, it must be pointed out that this method appears more robust with respect to estimation errors than the method which would consist in performing only one contrast optimization.

C. Extraction of the whole set of sources

So far, we have been concerned with the problem of extracting a single source from a mixture of N sources observed from Q sensors. Iterative solutions have been proposed to extract all sources. Among them, the so-called "deflation" method [33] consists in subtracting the contribution of the source which has been previously extracted from the sensors (see also [48, 42] for other details). This operation can be performed by a least square approach and then, the remaining problem amounts to separating $N-1$ sources from the Q observed signals. Repeating this operation iteratively leads to the extraction of all the N sources.

Let us now see the consequence of a "deflation" step on the matrix $\mathbf{R}_{\underline{\mathbf{x}}}$ which has been defined in Section IV-A and which is used for the extraction of a single source. Defining the $QD \times N(L+D-1)$ matrix

$$\mathcal{T}(\mathbf{M}) \triangleq \begin{pmatrix} \mathbf{M}(0) & \ldots & \mathbf{M}(L-1) & 0 & \ldots & 0 \\ 0 & \mathbf{M}(0) & \ldots & \mathbf{M}(L-1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \mathbf{M}(0) & \ldots & \mathbf{M}(L-1) \end{pmatrix}$$

and the column vector

$$\underline{\mathbf{s}}(n) \triangleq \begin{pmatrix} \mathbf{s}(n)^T & \mathbf{s}(n-1)^T & \ldots & \mathbf{s}(n-L-D+2) \end{pmatrix}^T,$$

we have $\underline{\mathbf{x}}(n) = \mathcal{T}(\mathbf{M})\underline{\mathbf{s}}(n)$ and hence

$$\mathbf{R}_{\underline{\mathbf{x}}} = \mathcal{T}(\mathbf{M})\mathbf{R}_{\underline{\mathbf{s}}}\mathcal{T}(\mathbf{M})^H \tag{37}$$

where $\mathbf{R}_{\underline{\mathbf{s}}} \triangleq \mathrm{E}\{\underline{\mathbf{s}}(n)\underline{\mathbf{s}}(n)^H\}$.

In addition to A5., we assume that the mixing system has strictly more sensors than sources ($Q > N$) and that the matrix $\mathcal{T}(\mathbf{M})$ is full column rank (that is in particular the case when the polynomial matrix $\mathbf{M}[z]$ is column reduced and when $D \geq N(L-1)$, see [32] for more details).

In this case, the rank of $\mathbf{R}_{\underline{\mathbf{x}}}$ is equal to the rank of $\mathbf{R}_{\underline{\mathbf{s}}}$. Thus, cancelling the contribution of one source on the sensors is equivalent to having one source that identically vanishes. Then, one can easily see that $L+D-1$ components of $\underline{\mathbf{s}}(n)$ identically vanish and therefore, the rank of $\mathbf{R}_{\underline{\mathbf{x}}}$ is reduced of the same number.

At the P-th source extraction stage, $P-1$ "deflation" steps have been performed and thus the rank of $\mathbf{R}_{\underline{\mathbf{x}}}$ has to be reduced by $(P-1)(L+D-1)$. This fact must be taken into account when projecting onto $(\ker \mathbf{R}_{\underline{\mathbf{x}}})^\perp$ as explained in Section IV-A. The procedure is summarized hereunder:

Let $\underline{\mathbf{x}}^0(n) = \underline{\mathbf{x}}(n)$. Estimate $r_0 = \mathrm{rank}(\mathbf{R}_{\underline{\mathbf{x}}^0})$ (or assume it is known). For $i = 1 \ldots N$ do:

1) decompose $\mathbf{R}_{\underline{\mathbf{x}}^{i-1}}$ according to (30), keeping the largest r_{i-1} eigenvalues. Then extract from $\mathbf{x}^{i-1}(n)$ a source according to the method from Section IV-B, so yielding $y_i(n)$.
2) if $i < N$, then perform a deflation step:
   - by a least square method, subtract $y_i(n)$ from $\underline{\mathbf{x}}^{i-1}(n)$, which yields $\underline{\mathbf{x}}^i(n)$.
   - Set $r_i = r_{i-1} - (P-1)(L+D-1)$.

V. Simulation results

The proposed approach is now illustrated through computer simulations. We have mainly considered real-valued source signals and mixing systems. Complex-valued signals and systems have also been tested because of their importance in telecommunication systems. In particular, non-linear CPM (Continuous Phase Modulation) source signals have been considered. If we except Experiment 3 where a comparison is performed between 3-rd and 4-th order based contrasts, the separation criterion which has been used is derived from 4-th order cumulants and is given by Equation (19).

All the results presented below result from Monte-Carlo simulations involving 100 realizations. At each run, the mixing system and the sources have been drawn randomly. The coefficients of the reference system have also been drawn randomly except in Section V-B. The coefficients of the mixing system and of the reference system have been generated according to a normal distribution.

A. Experiment 1 – MSE w.r.t. iteration number

Here, we consider a mixture of i.i.d. PAM-4 source signals, i.e. signals taking their values in the set $\{-\frac{3}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \frac{3}{\sqrt{5}}\}$ with equal probabilities. The length of the mixing filter is $L = 3$, the number of sources is $N = 3$ and the number of observations is $Q = 4$. The simulations have been carried out with $K = 5000$ and $K = 10000$ samples, successively. In Tables I and II, we give the average mean square error (MSE) on the estimated source signal MSE versus the number of iterations $\ell_{\max}$ used for the "fixed-point" method described in Section IV-B. In Table I, the initial reference system has been drawn randomly, whereas in Table II, it has been arbitrarily set to $\underline{\mathbf{w}}^0 = (1, 0, \ldots, 0)$.

One notices in both Tables I and II that after a few iterations, the MSE becomes constant: the validity of the method proposed in Section IV-B is confirmed by the low error values which are obtained. In addition, one may observe that the results become interesting even after the first iteration. The importance of the choice of the reference signal is investigated in the next paragraph.

B. Experiment 2 – Influence of the reference signal

We consider the case of $N = 2$ i.i.d. PAM-4 source signals, $Q = 3$ observation signals and a mixing filter of length $L = 3$. To evaluate the influence of the reference signal, we choose it as $z(n) = \beta s_1(n) + (1-\beta)s_2(n)$ with $\beta \in [0, 1]$. In so doing, we are able to investigate the sensitivity of the contrast function w.r.t. the choice of the reference system and the conditions in Proposition 1. The latter conditions are met, except when $\beta = 1/2$. In other words, this experiment is carried out in a semi-blind context where $z(n)$ is given and where the influence of its choice is evaluated. Incidently, the choice of the reference signal is linked to the closeness between the principal eigenvalue of the matrix $\tilde{\mathbf{C}}_{\underline{\mathbf{x}},\mathbf{z}}$ in Equation (36) and its next largest eigenvalue.

In Figure 1, the MSE of the estimated source signal is plotted versus β for three different sample sizes and for a constant number of iteration $\ell_{\max} = 1$. In Figure 2, the MSE of the estimated source signal is plotted versus β for four different iteration numbers and for a constant number of samples $K = 5000$.

As expected, better results are obtained for values of β near 0 or 1, that is when the reference signal is closer to one of the sources. Moreover, with a sufficient number of samples, satisfactory values of the MSE are observed, except for values of β close to $1/2$. By increasing the number of samples, this behaviour becomes more apparent. This confirms the fact that, when the exact statistics are available, the proposed criteria constitute contrasts, except for $\beta = 1/2$ (which arises with probability zero when the reference system is randomly driven).

C. Experiment 3 – Comparison with non quadratic contrasts

It has been well established that the square modulus of the fourth order cumulant $|\mathrm{C}_4\{y(n)\}|^2$ of the separator output is a contrast [48, 42]. We now compare this classical contrast with the quadratic contrast in Equation (19). The latter one has been used as described in Section IV-B with $\ell_{\max} = 5$ successive optimizations and an initial reference system randomly chosen. On the other hand, the contrast $|\mathrm{C}_4\{y(n)\}|^2$ has been optimized using a gradient ascent method with an adaptatively adjusted step-size (initially set to one and divided by two each time the contrast does not increase).

In these simulations we have considered $N = 3$ i.i.d. uniform source signals mixed on $Q = 4$ sensors by a filter of length $L = 3$. The number of samples has been set to $K = 5000$.

First and foremost, the quadratic contrasts proposed in this paper are optimized much quicker than the classical ones. The results in Table III show up that $\ell_{\max} = 5$ *optimizations of the quadratic contrast can be performed about 25 times faster than a single* optimization of a classical non quadratic contrast[2].

In Figure 3, the empirical MSE cumulative distribution function is also displayed. One can see that the "reference" based approach outperforms the classical one.

In conclusion, the quadratic contrasts appear as very interesting tools, both for their easy and fast optimization and for the good quality results they provide.

[2]The execution times have been estimated on a computer running a Pentium 4 processor running at 1GHz clock frequency and with 256MB RAM.

D. Experiment 4 – Comparison between 3-rd and 4-th order based contrasts

The contrast functions given by Equations (11) and (12) can be based on cumulants of any order greater than or equal to 3. Intuitively, the lower the order of the considered cumulants, the lower the estimation errors should be. We here illustrate this fact in the case of $N = 3$ skewed i.i.d. source signals taking their values in the set $\{-1, 0, \alpha\}$ with the respective probabilities $\{\frac{1}{1+\alpha}, \frac{\alpha-1}{\alpha}, \frac{1}{\alpha(\alpha+1)}\}$. The parameter α allows us to vary continuously the values of the cumulants of these source signals, here denoted by MS(α) [36]. In our experiments, we have used $\alpha = 3$ or $\alpha = 1 + \sqrt{2}$. The respective values of the third and fourth order cumulants are $\mathrm{C}_3\{s(n)\} = 2$, $\mathrm{C}_4\{s(n)\} = 4$ for $\alpha = 3$ and $\mathrm{C}_3\{s(n)\} = \mathrm{C}_4\{s(n)\} = \sqrt{2}$ for $\alpha = 1 + \sqrt{2}$. Finally, the length of the mixing filter was $L = 3$ and the number of observed signals was $Q = 4$.

In Figure 4, the MSE of the estimated source signal is plotted versus the number of samples. The results have been obtained with $\ell_{\max} = 3$ iterations as described in Section IV-B. As intuitively expected, we observe that the contrast function based on third order statistics leads to a better performance.

E. Experiment 5 – Non i.i.d. source signals

1) Considered signals and separation criterion: Different non i.i.d. source signals have been considered to illustrate the validity of the contrast functions in this context. Hereafter, we shortly describe the models we have used for the N sources $s_i(n), i \in \{1, \dots, N\}$.

- Multiplicative like models have been generated: each source reads $s_i(n) = a_i(n)\zeta_i(n)$ where $\zeta_i(n)$ is an i.i.d. normalized Gaussian process. When in addition $a_i(n)$ is a moving average (MA) time series, we say that the sources are "stochastic volatility" processes [5]. When $a_i(n)$ is positive and satisfies the following "auto-regressive" equation:

$$a_i(n)^2 = \alpha_0 + \sum_{k=1}^{P} \alpha_k s_i(n-k)^2 \tag{38}$$

  we obtain the so-called ARCH(P) model (autoregressive conditionnal heteroscedastic of order P, see e.g. [41]).
- Pseudo-symbols of Continuous Phase Modulated signals are unit modulus signals defined by the relation $s_i(n+1) = \exp(\imath\pi h_i a_n^i)s_i(n)$ where $h_i \in]0, 1[$ is a modulation index and a_n^i is the i.i.d. binary symbol sequence transmitted by the i-th user.

Since non i.i.d. sources can be recovered only up to a scalar filtering, we introduce the following separation criterion:

$$\tau \triangleq 1 - \frac{\max_j \|g_j\|_j^2}{\sum_{j=1}^{N} \|g_j\|_j^2} \tag{39}$$

One can notice that $0 \leq \tau < 1$ and, according to Definition 2, we have $\tau = 0$ if and only if one source is perfectly extracted.

2) Simulation results: We have considered the case of $N = 3$ non i.i.d. source signals, $Q = 4$ observation signals and a mixing filter of length $L = 3$. Figure 5 represents the average value of τ obtained for the first source which has been extracted in the same way as in Section V-A with $\ell_{\max} = 3$ optimizations of quadratic contrasts and a separator of length $D = 6$. These results are provided for mixtures obtained from sources of the three types described above. We observe that the separation is sucessful for all kind of sources. This suggests that the condition from Proposition 5 may be not too restrictive for many source models, although it may be difficult to check theoretically.

Table IV is similar to Table II: the criterion τ is given with respect to the number of successive optimizations for 1000 and 5000 available samples of CPM sources. Again, we obtain a good separation performance after a few iterations. We also report in Table V the influence of the length of the separator for a fixed number of sources ($N = 3$) and length of the mixing filter ($L = 3$). Interestingly, the method seems quite robust with respect to an overestimation of the length of the separator.

We have studied the influence of the number of sources and of the length of the mixing filter in the case of CPM sources (depending on the number of sources, the modulation indices have been chosen in $\{0.2; 0.3; 0.4; 0.6; 0.7\}$). The values of the criterion τ stem from an average on 100 realizations and are given in Figure 6. The performance is particularly good when the number of sources or the filter length is small. As classically observed, the performance declines when either of these quantities grows up.

F. Experiment 6 – The MIMO case using a deflation procedure

Using the proposed quadratic contrasts, we have tested a separation procedure which extract all the sources. The method is based on a deflation procedure and takes into account the details given in Section IV-C.

We have considered $N = 3$ source signals mixed on $Q = 6$ sensors with a filter of length $L = 3$. The number of iterations (i.e. successive optimizations) for each source extraction was $\ell_{\max} = 5$. In Table VI (resp. Table VII) the case of i.i.d. PAM4 (resp. non i.i.d.) source signals is considered. In Table VII, τ_1, τ_2 and τ_3 respectively denote the value of the criterion defined in Equation (39) for the 1-st, 2-nd and 3-rd source. (Of course, the maximum on j in (39) should not obtained for the same value of j for two different criteria τ_i and $\tau_{i'}$ with $i \neq i'$ since this would mean that the same source would have been recovered twice: we have introduced this constraint in the way we calculate successively τ_2 and τ_3). As classically observed in deflation separation methods, the performance is better for the first extracted source signal. However, it appears to remain quite satisfactory for the 3 extracted sources. More importantly, the experiments in this subsection confirm the validity of the comments made in Section IV-C.

VI. CONCLUSION

Starting from the idea of exploiting reference signals, we have proposed quadratic contrasts. We have first proved their validity under some necessary conditions for the reference system. Noticing how mild these conditions are, we have shown that quadratic contrast functions may be used even in a totally blind scenario. These conclusions hold for both i.i.d. and non i.i.d. sources.

As shown by our simulation results, one may be impressed by the capabilities of these new contrast functions. They indeed offer a performance which is, at least, as good as other cumulant based contrasts like the absolute value of the kurtosis. Besides, their fast optimization capabilities make them extremely competitive compared with other existing approaches.

Concrete implementation issues have been discussed in the paper. As explained, some details have to be taken into account when one wants to combine these quadratic contrasts with a deflation procedure. All elements are hence available to the reader for the implementation of the proposed separation method[3].

Appendix I
Proof of Proposition 1

For the sake of clarity, we will give the proof only for the criterion $\mathcal{C}_{R,\mathbf{z}}\{y(n)\}$ derived from (11) where $y^{(*)_1}(n) = y^{(*)_2}(n) = y(n)$. It can be easily adapted to other cases. Assuming that the conditions in Section III-C hold for the references $\mathbf{z}$ (that is $\mathbf{z}$ is obtained by a filtering of the sources: see Eq. (16)), we can write

$$\kappa_{R,\mathbf{z}}\{y(n)\} = \sum_{j=1}^{N}\sum_{k\in\mathbb{Z}} g_j(k)^2 \kappa_{R,\mathbf{z}}\{s_j(n-k)\} \tag{40}$$

and, using (13) and the unit-norm property of $\mathbf{g}(n)$, it follows

$$\mathcal{C}_{R,\mathbf{z}}\{y(n)\} \le \sum_{j=1}^{N}\sum_{k\in\mathbb{Z}} |g_j(k)|^2 |\kappa_{R,\mathbf{z}}\{s_j(n-k)\}| \tag{41}$$

$$\le \kappa_R^{\max} \sum_{j=1}^{N}\sum_{k\in\mathbb{Z}} |g_j(k)|^2 = \kappa_R^{\max}\ . \tag{42}$$

Recalling (14), property p1 in Definition 1 follows easily. Whenever the above upper-bound is reached, which is possible according to Assumption A2., then

$$\sum_{j=1}^{N}\sum_{k\in\mathbb{Z}} |g_j(k)|^2 \big(\kappa_R^{\max} - |\kappa_{R,\mathbf{z}}\{s_j(n-k)\}|\big) = 0. \tag{43}$$

All the terms in the above summation being positive, if $\mathcal{I}$ contains a single element (i_0, k_0), one deduces that the global Multi-Input / Single-Output filter $\{\mathbf{g}\}$ satisfies the equalization condition (5). Hence Property p2 in Definition 1 holds whenever $\mathcal{I}$ contains a single element. Conversely, one can see that if $\mathcal{I}$ contains several elements, say $(j_1, k_1) \in \mathcal{I}$ and $(j_2, k_2) \in \mathcal{I}$, the filter given by $g_{j_1}(k_1) = g_{j_2}(k_2) = \frac{1}{\sqrt{2}}$ and $g_j(k) = 0$ otherwise, maximizes $\mathcal{C}_{R,\mathbf{z}}$ although it is not separating. Hence there exist non separating filters which maximize $\mathcal{C}_{R,\mathbf{z}}$ and Property p2 in Definition 1 does not hold any more. ■

[3]The Matlab code used for our simulations are available at the address `http://www-syscom.univ-mlv.fr/toolbox/`.

Appendix II
Proof of Proposition 4

The reader should keep in mind that in a blind context, non i.i.d. sources can only be recovered up to a scalar filtering ambiguity. Hence, similarly to [42], we will consider the norms of the components of the global filter and prove that only one component is non zero. For any $j \in \{1,\dots,N\}$, we write the j-th component of the filter $\{\mathbf{g}\}$ as $\{g_j\} = \|g_j\|_j\{\widetilde{g}_j\}$ where $\{\widetilde{g}_j\}$ equals $\{g_j\}/\|g_j\|_j$ if $\|g_j\|_j \neq 0$ and $\{\widetilde{g}_j\}$ identically vanishes if $\|g_j\|_j = 0$. Defining $\widetilde{y}_j(n) \triangleq \{\widetilde{g}_j\}s_j(n)$, we obtain[4] by cumulant multilinearity and mutual independence of the signals $\widetilde{y}_j(n)$, $j \in \{1,\dots,N\}$:

$$\kappa_{R,\mathbf{z}}\{y(n)\} = \sum_{j=1}^{N} \|g_j\|_j^2 \kappa_{R,\mathbf{z}}\{\widetilde{y}_j(n)\}. \tag{44}$$

Therefore, we have the following inequality:

$$\mathcal{C}_{R,\mathbf{z}}\{y(n)\} \le \sum_{j=1}^{N} \|g_j\|_j^2 \mathcal{C}_{R,\mathbf{z}}\{\widetilde{y}_j(n)\}. \tag{45}$$

Recalling (20) and the fact that $\|\widetilde{g}_j\|_j = 1$, this yields:

$$\mathcal{C}_{R,\mathbf{z}}\{y(n)\} \le \sum_{j=1}^{N} \|g_j\|_j^2 \mathcal{M}_j \le \mathcal{M}_{\max} \sum_{j=1}^{N} \|g_j\|_j^2 \le \mathcal{M}_{\max}.$$

The above inequality proves property p1' in Definition 2. In addition, if equality holds, we can write

$$\sum_{j=1}^{N} \|g_j\|_j^2 \big(\mathcal{M}_{\max} - \mathcal{M}_j\big) = 0 \tag{46}$$

from which we deduce that for all $j \in \{1,\dots,N\}$, we have either $\|g_j\|_j = 0$ or $\mathcal{M}_j = \mathcal{M}_{\max}$. Hence, see that property p2' in Definition 2 is satisfied if $\mathcal{I}'$ contains one element only. Conversely, if $\mathcal{I}'$ contains several elements, one can easily see that the maximum value of the criterion may be reached by a non separating filter. Proposition 4 follows. ■

Appendix III
Proof of Proposition 5

In order to simplify notations, the proof will be provided only for the contrast given by Equation (19) which is derived from fourth-order cumulants ($R = 4$). It can obviously be adapted to other cases. The proof has been split into three parts: first we find a lower-bound for $\mathcal{M}_j$, then we find an upper-bound and finally we conclude.

[4] Similarly to Eq. (40), we implicitely assume in Eq. (44) that the conditions in Section III-C hold for the references $\mathbf{z}$ (that is $\mathbf{z}$ is obtained by a filtering of the sources: see Eq. (16)).

A. Lower-bound

Using cumulant multilinearity and the independence of the sources, we have:

$$\begin{aligned}\kappa_{4,\mathbf{z}}\{\tilde{y}_j(n)\} = & \sum_{k_1,k_2,l_1,l_2} \tilde{g}_j(k_1)\tilde{g}_j^*(k_2)t_j(l_1)t_j^*(l_2) \\ & \mathrm{Cum}\{s_j(n-k_1), s_j^*(n-k_2), s_j(n-l_1), s_j^*(n-l_2)\} \\ = & \sum_{k_1,k_2,l_1,l_2} \tilde{g}_j(k_1)\tilde{g}_j^*(k_2)t_j(l_1)t_j^*(l_2) \\ & \mathrm{C}_4^{(k_1-k_2,k_1-l_1,k_1-l_2)}\{s_j(n)\}.\end{aligned} \tag{47}$$

Reminding (20), for any $p \in \mathbb{Z}$ we can write the following inequality, where we have considered the particular case $\tilde{g}_j(n) = \delta_{n-p}$:

$$\begin{aligned}\mathcal{M}_j \geq & \Big| \sum_{k_1,k_2,l_1,l_2} \delta(k_1-p)\delta(k_2-p)t_j(l_1)t_j^*(l_2) \\ & \mathrm{C}_4^{(k_1-k_2,k_1-l_1,k_1-l_2)}\{s_j(n)\}\Big| \\ = & \Big|\sum_{l_1,l_2} t_j(l_1)t_j^*(l_2)\mathrm{C}_4^{(0,p-l_1,p-l_2)}\{s_j(n)\}\Big| \\ \geq & |t_j(p)|^2|\mathrm{C}_4^{(0,0,0)}\{s_j(n)\}| \\ & - \Big| \sum_{(l_1,l_2)\neq(p,p)} t_j(l_1)t_j^*(l_2)\mathrm{C}_4^{(0,p-l_1,p-l_2)}\{s_j(n)\}\Big|.\end{aligned}$$

In addition, we have:

$$\begin{aligned}& \Big| \sum_{(l_1,l_2)\neq(p,p)} t_j(l_1)t_j^*(l_2)\mathrm{C}_4^{(0,p-l_1,p-l_2)}\{s_j(n)\}\Big| \leq \\ & \sup_{(l_1,l_2)\neq(p,p)} |\mathrm{C}_4^{(0,p-l_1,p-l_2)}\{s_j(n)\}| \sum_{(l_1,l_2)\neq(p,p)} |t_j(l_1)||t_j(l_2)| \\ & = \sup_{(l_1,l_2)\neq(0,0)} |\mathrm{C}_4^{(0,l_1,l_2)}\{s_j(n)\}|(\|t_j\|_{\ell_1}^2 - |t_j(p)|^2).\end{aligned}$$

Combining the previous two inequalities and taking the supremum over $|t_j(p)|, p \in \mathbb{Z}$ we obtain:

$$\begin{aligned}\mathcal{M}_j \geq & \|t_j\|_{\ell_\infty}^2 \Big(|\mathrm{C}_4^{(0,0,0)}\{s_j(n)\}| \\ & + \sup_{(l_1,l_2)\neq(0,0)} |\mathrm{C}_4^{(0,l_1,l_2)}\{s_j(n)\}|\Big) \\ & - \|t_j\|_{\ell_1}^2 \sup_{(l_1,l_2)\neq(0,0)} |\mathrm{C}_4^{(0,l_1,l_2)}\{s_j(n)\}|.\end{aligned} \tag{48}$$

B. Upper-bound

According to (47), we can write:

$$\begin{aligned}& |\kappa_{4,\mathbf{z}}\{\tilde{y}_j(n)\}| \\ & \leq \sum_{k_1,k_2,l_1,l_2} |\tilde{g}_j(k_1)||\tilde{g}_j^*(k_2)||t_j(l_1)||t_j^*(l_2)| \\ & \qquad |\mathrm{C}_4^{(k_1-k_2,k_1-l_1,k_1-l_2)}\{s_j(n)\}| \\ & \leq \|t_j\|_{\ell_\infty}^2 \sum_{k_1,k_2} |\tilde{g}_j(k_1)||\tilde{g}_j(k_2)| \sum_{l_1,l_2} |\mathrm{C}_4^{(k_1-k_2,l_1,l_2)}\{s_j(n)\}| \\ & \leq \|t_j\|_{\ell_\infty}^2 \sum_{l_1,l_2}\sum_p |\mathrm{C}_4^{(p,l_1,l_2)}\{s_j(n)\}| \sum_k |\tilde{g}_j(k)||\tilde{g}_j(k-p)| \\ & \leq \|t_j\|_{\ell_\infty}^2 \sum_{l_1,l_2}\sum_p |\mathrm{C}_4^{(p,l_1,l_2)}\{s_j(n)\}| \sum_k |\tilde{g}_j(k)|^2\end{aligned}$$

where in the last step we have used Cauchy-Schwarz inequality. We next note that, according to Parseval relation,

$$\begin{aligned}\|\tilde{g}_j\|_j^2 = & \sum_{(k_1,k_2)\in\mathbb{Z}^2} \tilde{g}_j^*(k_1)\tilde{g}_j^*(k_2)\gamma_j(k_2-k_1) \\ & = \int_{-1/2}^{1/2} |\tilde{G}_j(f)|^2\Gamma_j(f)\,df \\ & \geq \Gamma_j^{\min}\int_{-1/2}^{1/2} |\tilde{G}_j(f)|^2\,df = \Gamma_j^{\min}\sum_k |\tilde{g}_j(k)|^2\end{aligned}$$

where $\tilde{G}_j(f)$ is the frequency response of the filter $\{\tilde{g}_j\}$ and $\Gamma_j q(f)$ is the j-th source spectrum density. We deduce that

$$\begin{aligned}\mathcal{M}_j \leq & (\Gamma_j^{\min})^{-1}\|t_j\|_{\ell_\infty}^2 \sum_{l_1,l_2}\sum_p |\mathrm{C}_4^{(p,l_1,l_2)}\{s_j(n)\}| \\ & = (\Gamma_j^{\min})^{-1}\|t_j\|_{\ell_\infty}^2 \|\mathrm{C}_4^{(\cdot)}\{s_j(n)\}\|_{\ell_1}.\end{aligned}$$

C. Conclusion of the proof

From the above upper and lower bounds, one can see that the following condition is sufficient to ensure that $\mathcal{M}_j < \mathcal{M}_{j_0}$ and hence to guarantee that the set $\mathcal{I}'$ in Equation (21) contains a single element $\{j_0\}$:

$$\begin{aligned}& (\Gamma_j^{\min})^{-1}\|t_j\|_{\ell_\infty}^2 \|\mathrm{C}_4^{(\cdot)}\{s_j(n)\}\|_{\ell_1} < \\ & \|t_{j_0}\|_{\ell_\infty}^2 \Big(|\mathrm{C}_4^{(0,0,0)}\{s_j(n)\}| + \sup_{(l_1,l_2)\neq(0,0)} |\mathrm{C}_4^{(0,l_1,l_2)}\{s_j(n)\}|\Big) \\ & \qquad - \|t_{j_0}\|_{\ell_1}^2 \sup_{(l_1,l_2)\neq(0,0)} |\mathrm{C}_4^{(0,l_1,l_2)}\{s_j(n)\}|\end{aligned}$$

This completes the proof of Proposition 5. ■

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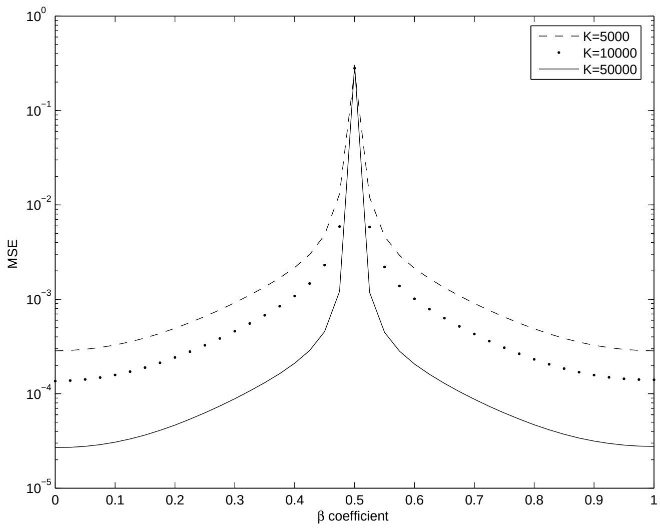

Fig. 1. MSE versus coefficient β (where the reference signal is $z(n) = \beta s_1(n) + (1-\beta)s_2(n)$) for different number of samples.

$\ell_{\max}$	1	2	3	4	5
$K = 5000$ samples	2.56×10^{-2}	8.32×10^{-4}	5.32×10^{-4}	5.15×10^{-4}	5.14×10^{-4}
$K = 10000$ samples	1.72×10^{-2}	5.22×10^{-4}	3.48×10^{-4}	3.37×10^{-4}	3.35×10^{-4}

TABLE I

EXTRACTION OF ONE SOURCE: AVERAGE MSE VERSUS NUMBER OF ITERATIONS. THE REFERENCE SYSTEM HAS BEEN RANDOMLY INITIALIZED.

$\ell_{\max}$	1	2	3	4	5
$K = 5000$ samples	8.22×10^{-2}	1.8×10^{-2}	6.36×10^{-4}	6.167×10^{-4}	6.163×10^{-4}
$K = 10000$ samples	4.36×10^{-2}	1.2×10^{-2}	2.95×10^{-4}	2.885×10^{-4}	2.882×10^{-4}

TABLE II

EXTRACTION OF ONE SOURCE: AVERAGE MSE VERSUS NUMBER OF ITERATIONS. THE REFERENCE SIGNAL IS ONE OBSERVED SIGNAL.

$\ell_{\max}$	1	2	3	4	5
$K = 1000$ samples	6.20×10^{-2}	5.3×10^{-3}	7.8×10^{-4}	5.33×10^{-4}	5.24×10^{-4}
$K = 5000$ samples	3.15×10^{-2}	3.0×10^{-3}	7.14×10^{-5}	3.3×10^{-5}	2.8×10^{-5}

TABLE IV

EXTRACTION OF ONE SOURCE FROM A MIXTURE OF 3 CPM SOURCES WITH INDICES 0.4, 0.75 AND 0.25 : τ VERSUS NUMBER OF ITERATIONS ($Q = 4$, MEDIAN VALUE).

D	6	16	26	36	46	56	66
$K = 10000$ samples	2.57×10^{-6}	6.21×10^{-6}	1.03×10^{-5}	1.09×10^{-5}	1.52×10^{-5}	1.84×10^{-5}	2.1×10^{-5}
$K = 15000$ samples	1.29×10^{-6}	4.29×10^{-6}	4.31×10^{-6}	8.61×10^{-6}	9.44×10^{-6}	9.66×10^{-6}	1.19×10^{-5}

TABLE V

EXTRACTION OF ONE SOURCE FROM A MIXTURE OF 3 CPM SOURCES WITH INDICES 0.4, 0.7 AND 0.3: INFLUENCE OF THE LENGTH D OF THE SEPARATING FILTER ON THE SEPARATION CRITERION τ ($Q = 4$, $\ell_{\max} = 5$, MEDIAN VALUE).

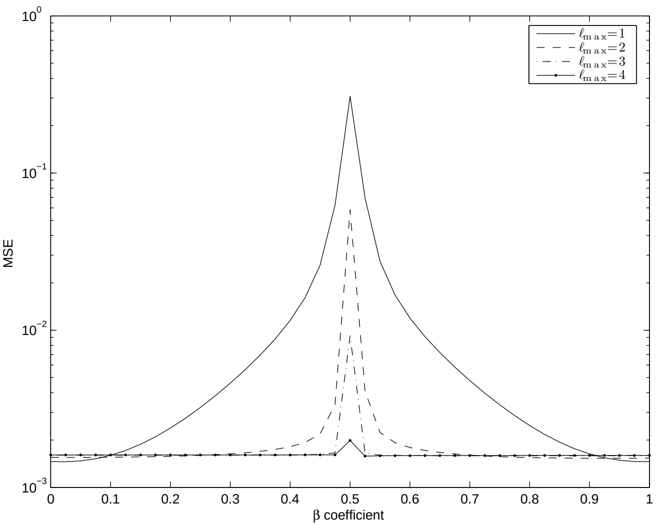

Fig. 2. MSE versus coefficient β (where the reference signal is $z(n) = \beta s_1(n) + (1 - \beta) s_2(n)$) for different number of iterations.

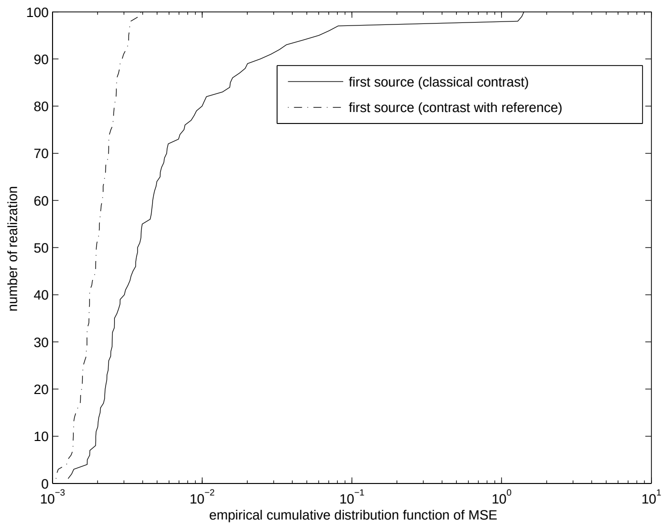

Fig. 3. For uniform source signals, comparison between MISO results using classical contrast and contrast with reference.

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Number of samples	5000	10000	25000	50000
1-st source	3×10^{-4}	1.38×10^{-4}	1.25×10^{-4}	3.12×10^{-5}
2-nd source	4.9×10^{-3}	1.4×10^{-3}	4.71×10^{-4}	5.25×10^{-4}
3-rd source	6.0×10^{-3}	1.6×10^{-3}	1.194×10^{-3}	6.66×10^{-4}

TABLE VI

MIMO SEPARATION OF 3 I.I.D. PAM4 SOURCE SIGNALS (AVERAGE MSE).

Number of samples		1000	5000	10000	25000
ARCH(1) sources	τ_1	1.181×10^{-1}	1.81×10^{-2}	1.59×10^{-2}	8.9×10^{-3}
	τ_2	1.541×10^{-1}	3.37×10^{-2}	2.94×10^{-2}	1.39×10^{-2}
	τ_3	1.524×10^{-1}	5.96×10^{-2}	3.34×10^{-2}	2×10^{-2}
CPM sources (modulation indices: 0.4, 0.75 and 0.25)	τ_1	1×10^{-4}	9×10^{-6}	5.6×10^{-6}	2.5×10^{-6}
	τ_2	1.22×10^{-2}	2.3×10^{-3}	1.9×10^{-3}	4×10^{-4}
	τ_3	2.59×10^{-2}	7.4×10^{-3}	5.2×10^{-3}	1×10^{-3}

TABLE VII

MIMO SEPARATION RESULTS WITH NON I.I.D. SOURCE SIGNALS (MEDIAN VALUE).

Number of samples	1000	5000	10000	25000
Contrast $\lvert \mathrm{C}_4\{y(n)\}\rvert^2$	32.10	211.06	418.59	784.61
Quad. Contrasts (5 iterations)	1.23	7.39	13.46	37.01

TABLE III

COMPARISON OF THE AVERAGE EXECUTION TIME (IN S) FOR: 1) ONE OPTIMIZATION OF THE CONTRAST $\lvert \mathrm{C}_4\{y(n)\}\rvert^2$ — 2) $\ell_{\max} = 5$ OPTIMIZATIONS OF QUADRATIC CONTRASTS.

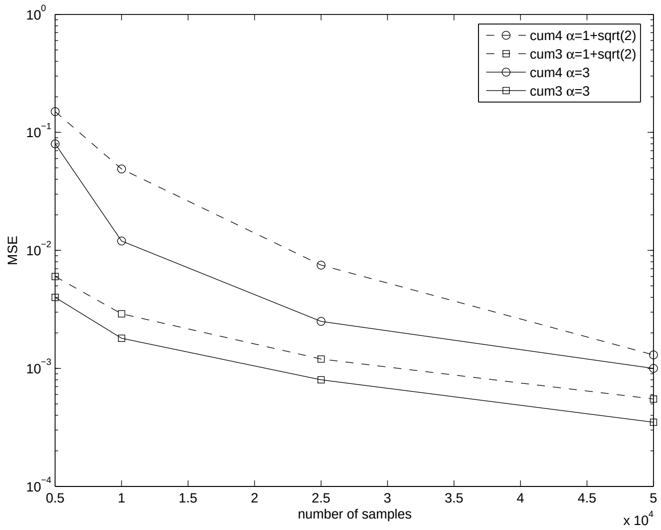

Fig. 4. Average MSE versus number of samples considering contrasts based on third and fourth order cumulants.

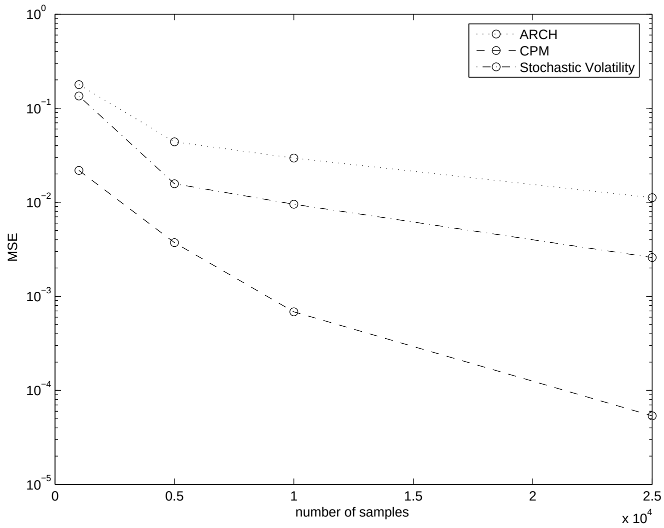

Fig. 5. Average separation criterion τ after extraction of one non i.i.d. source (characteristics of the sources: ARCH(3) / CPM with indices 0.4, 0.75, 0.25 / stochastic volatility with a MA(3) multiplicative process).

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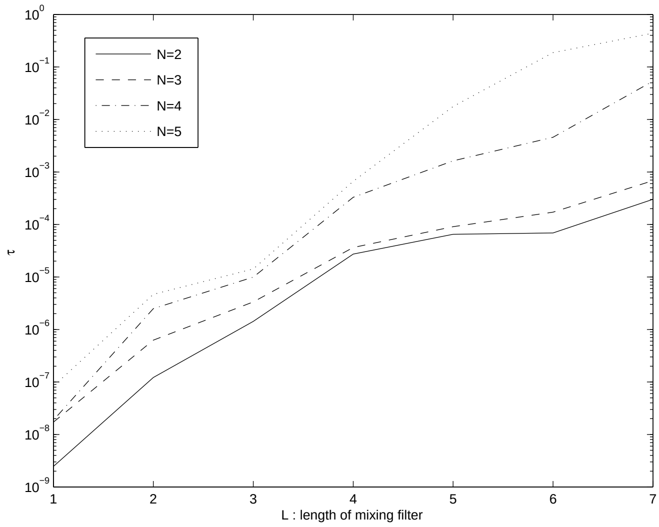

Fig. 6. Influence of the number of sources and the length of the mixing filter on the separation criterion τ (CPM sources, number of sensors set to $Q = N + 1$, $\ell_{\max} = 5$ successive optimizations, 20000 samples).

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