

# CSC 4504 : Langages formels et applications

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## **The MUI TRS**

# Typographical/term Re-write Systems (TRS)

A TRS is a formal system based on the ability to generate a set of strings following a simple set of syntactic rules.

Each rule is calculable --- the generation of a new string from an old string by application of a rule always terminates

A TRS may produce an infinite number of strings

TRSs can be as powerful as any computing machine (Turing equivalent)

TRSs are simple to implement (simulate) using other computational models

Using TRSs we introduce the following concepts:

*proof, theorem, decision procedure, meta-analysis, structural induction, necessary and sufficient, isomorphism, meaning and consistency*

Don't worry ... they are very simple to understand ....

# Case Study 1 --- The MUI TRS

## Taken from:

Hofstadter, Douglas H. *Gödel, Escher, Bach: An Eternal Golden Braid; [a Metaphoric Fugue on Minds and Machines in the Spirit of Lewis Carroll]*. Penguin Books, 1980.



# Case Study 1 --- The MUI TRS

*Alphabet* = {M,I,U}

**Strings:** any sequence of characters found in the alphabet

**Axiom:** MI

**Generation Rules:** for all strings such that x and y are (possibly empty) strings of *Alphabet*:

- 1) xI can generate xIU
- 2) Mx can generate Mxx
- 3) xIIly can generate xUy
- 4) xUUy can generate xy

A **theorem** of a TRS is any string which can be generated from the axioms (or any other theorem)

A **proof** of a theorem corresponds to the set of rules which have been followed to generate that theorem

For example, **MIUIU** is a theorem, (proof: Axiom: 1,2)

# Case Study 1 --- The MUI TRS (proof procedure)

*Alphabet* = {M,I,U}

**Strings:** any sequence of characters found in the alphabet

**Axiom:** MI

**Generation Rules:** for all strings such that x and y are (possibly empty) strings of *Alphabet*:

- 1) xI can generate xIU
- 2) Mx can generate Mxx
- 3) xIIIy can generate xUy
- 4) xUUy can generate xy

**Question 1:** can you prove the theorem MUIIU?

**Question 2:** is IIIIUUUUIIIUUUI a theorem of the system?

**Question 3:** is MU a theorem of the system?