CSC 4504 : Langages formels et applications

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The pq-TRS
Problem 2 --- The pq- TRS

Alphabet = \{p,q,-\}

Axiom: for any such x such that x is a possibly empty sequence of ‘-’s,

\[ xp-qx- \] is an axiom

Generation Rules: for any x,y,z which are possibly empty sequences of ‘-’s, if \( xpyqz \) is a theorem then \( xpy-qz- \) is a theorem

A decision procedure for a TRS is a process that will always terminate with the correct answer (of whether a given string is a theorem or not

Question: is there a decision procedure for this formal system?
Problem 2 --- The pq- TRS

Alphabet = \{p, q, -\}

Axiom: for any such x such that x is a possibly empty sequence of ‘-’s,

xp-qx- is an axiom

Generation Rules: for any x, y, z which are possibly empty sequences of ‘-’s,
if xpyqz is a theorem then xpy-qz- is a theorem

Why is the pq- TRS practical?

Because it provides us with a formal model of a mathematical property: the addition of integers ---

• --p--q----- is a theorem and “2+3=5” is true
• --p-q-- is a non-theorem and “2+1=2” is false
Problem 2 --- The pq- TRS interpretation

If we interpret

• p as plus
• q as equals

• and a sequence of n ‘-’s as the integer n

then we have

a means of checking \( x+y=z \) for all non-negative integers \( x, y \) and \( z \)

We say that pq- is **consistent** (under the given interpretation) because all theorems are true after interpretation.

We say that pq- is **complete** as all true statements (in the domain of interpretation) can be generated as theorems in the system.

We say that the interpretation is **isomorphic** to the system because it is both complete and consistent.
Problem 2 --- The pq- TRS extension

The pq- system is isomorphic to a very limited domain of interpretation (but maybe that is all that is required!)

Normally, to widen a domain we can

   add an axiom

   add a generating rule

For example, what happens if we add the axiom:

   xp− qx.

Using this, we can generate many new theorems!

**Question**: with this new axiom what about completeness and consistency? Can you find an isomorphic interpretation?
Problem 2 --- The tq- system

Question:

• can you define a TRS for modelling the multiplication of two integers

• can you show that it is complete and consistent

Interpretation:

• t as times

• q as equals

• sequences of ‘-’s as integers