Performance analysis of a new calibration method for fiber nonlinearity compensation

Frederic Lehmann and Yann Frignac

***Abstract*—Digital signal processing for fiber nonlinearity compensation is a key enabler for the ever-increasing demand for higher data rates in coherent optical transmissions. A major challenge of existing techniques is that the fiber nonlinear coefficient needs to be scaled properly during compensation in order to reach the achievable signal quality increase. We solve this problem using a low-complexity algorithm adaptively optimizing a metric based on the soft-decision bitwise demodulator used for modern FEC decoders. An analytical model shows that the proposed scheme converges to the optimal scaling factor with a predictable precision, that is validated by numerical results.**

***Index Terms*—Coherent optical communications, soft demodulation, nonlinearity compensation.**

I. Introduction

Commercially available advanced fiber-optic communication systems achieve high spectral efficiency thanks to the application of modern digital signal processing (DSP) at the receiver to compensate linear channel impairments, such as chromatic dispersion (CD) and polarization mode dispersion (PMD) [1]. Higher data rates can be reached by increasing the fiber launch power, thus giving rise to nonlinear distorsions induced by the Kerr nonlinearity coefficient of the propagation medium, γ [2]. Thus, nonlinear fiber distorsions remain the dominant impediment to the ever growing demand for data traffic [2]. During the last decade, DSP methods have been developped to tackle this problem at the receiver side, including the inverse Volterra series transfer function (IVSTF) based nonlinear equalizer [3] and digital back-propagation (DBP), a method inverting the propagation equations using the split-step Fourier method (SSFM) [4] (see also its simplified perturbation [5] and lowpass-filtered [6] variations). A common feature of the aforementioned state-of-the art methods is the necessity to estimate a correction factor that linearly scales γ, in order to optimize the averaged signal quality (Q^2-factor), that relates to the bit-error rate (BER) (see Sec. V-A). The reason is that in the IVSTF method, the signal power must be locked to the receiver sensitivity [3], while in DBP and its variants the number of SSFM steps is finite. Several methods to adjust the scaling factor are available in the literature. An offline grid search, monitoring the Q^2-factor is presented in [4], but it requires prior knowledge of the transmitted data. In [6], an analytical formula is introduced, but accurate knowledge of the physical parameters of the transmission link is necessary. While the aforementioned methods are suitable for laboratory experiments, in real applications, the nonlinear coefficient of the fiber may vary in each span, or even be unknown. This motivates the search for online estimation methods. The principle of an adaptive semi-blind approach based on monitoring the signal quality was introduced in [7]. A practical example of semi-blind Q^2-factor estimator, based on the decision regions of the signal constellation, was suggested in [8]. However, this method is prone to large overestimation caused by hard decision errors in the presence of background noise and phase noise.

In this letter, we first introduce a new metric to monitor the signal quality with a low hardware complexity increase, while staying immune to hard decision errors irrespective of the modulation format. Then, we aim to optimize the nonlinearity scaling factor by minimization of the proposed metric using a stochastic gradient-descent. Finally, we characterize the convergence and the residual estimation error of the proposed scheme. Numerical results confirm the predicted performance.

We use $E[.]$ to denote the expectation operator.

II. System model and problem formulation

We consider the optical transmission system depicted in Fig. 1. At the transmitter side, we assume that a modern soft-decision forward error correction (FEC) encoder is used [9]. The i-th binary codeword $\mathbf{c}_i = (c_i^1, \ldots, c_i^N)$ at the output of the FEC encoder is mapped to elements of a size-M complex constellation, before pulse shaping and optical modulation.

At the receiver side, after the analog front-end, a digital coherent receiver performs polarization demultiplexing using an adaptive filter, followed by carrier recovery, soft demodulation and soft-decision bit-wise FEC decoding [1]. Unlike conventional receivers [1], where chromatic dispersion (CD) is compensated through linear equalization, joint CD and nonlinearity (NL) compensation (CD/NL-C) is applied [3]-[4]. Note that in order to obtain the maximum signal quality, CD/NL-C requires to apply a correction factor κ to the fiber nonlinearity parameter [7]. Since the optimal value of κ, denoted by κ_{opt}, is in general not known a priori, in practice CD/NL-C uses an estimate $\hat{\kappa}_i$ instead, that is fixed over the duration of the i-th codeword.

III. Proposed correction factor adjustment

We seek a metric related to the signal quality, that is immune to hard decision errors unlike the Q^2-factor estimator in [8]. Let $p_i^k(\kappa)$ be the *a posteriori* probability that the k-th bit within the i-th codeword is a binary zero (see Eq. (5) in [10]), when the CD/NL-C correction factor is fixed to κ.

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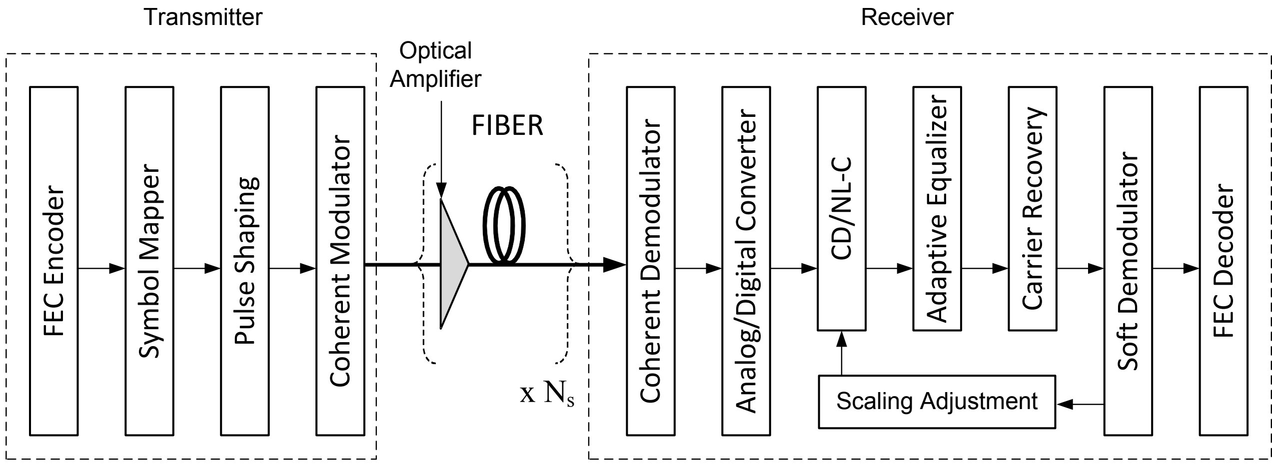

Fig. 1. Block diagram of the optical fiber transceiver. CD/NL-C: chromatic dispersion and nonlinearity compensator. N_s: number of fiber spans.

Since the computation of $p_i^k(\kappa)$ solely depends on the likelihood function of a received symbol, no hard decision error is incurred. Let us define the *a posteriori* average entropy [11] over the i-th codeword as

$$E_i(\kappa) = \frac{1}{N}\sum_{k=1}^{N} H_2(p_i^k(\kappa)), \tag{1}$$

where $H_2(x) = -x\log_2 x - (1-x)\log_2(1-x)$ is the binary entropy function. The metric (1) gives a measure of the reliability of the decisions at the soft demodulator output, applicable for any modulation format. A suitable objective function that reaches its minimum at $\kappa = \kappa_{\mathsf{opt}}$ is $J(\kappa) = E[E_i(\kappa)]$, where the expectation is taken with respect to the data, background noise, local oscillator (LO) and nonlinear phase noise [1]. A tractable adaptive estimation algorithm is obtained using a stochastic gradient-descent with step-size μ

$$\hat{\kappa}_i = \hat{\kappa}_{i-1} - \mu \frac{\partial E_i(\kappa)}{\partial \kappa}\Big|_{\kappa=\hat{\kappa}_{i-1}}. \tag{2}$$

A practical algorithm is obtained by replacing the partial derivative with a finite difference,

$$D_i = \frac{E_i(\hat{\kappa}_{i-1}+h) - E_i(\hat{\kappa}_{i-1}-h)}{2h}, \tag{3}$$

where $h > 0$ is a design parameter.

IV. Theoretical performance analysis

The finite difference term can be decomposed as

$$D_i = \frac{J(\hat{\kappa}_{i-1}+h) - J(\hat{\kappa}_{i-1}-h)}{2h} + N_i, \tag{4}$$

where the first term is the expected value. N_i represents the zero-mean perturbation term originating from the joint effect of the data, background noise, LO and nonlinear phase noise trajectory during the i-th codeword. Therefore, the perturbation terms for successive codewords can reasonably be assumed to be independently distributed. Linearizing (4) around $\kappa = \kappa_{\mathsf{opt}}$ using a second-order Taylor expansion, we obtain

$$D_i \approx A(\hat{\kappa}_{i-1} - \kappa_{\mathsf{opt}}) + N_i, \tag{5}$$

where

$$A = \frac{\partial^2 J(\kappa)}{\partial \kappa^2}\Big|_{\kappa=\kappa_{\mathsf{opt}}}. \tag{6}$$

Also, due to the independence assumption

$$E[N_i N_j] = \sigma_N^2 \delta_{i,j}, \tag{7}$$

where σ_N^2 is the variance of the perturbation at $\kappa = \kappa_{\mathsf{opt}}$ and $\delta_{i,j}$ is the Kronecker delta function.

Let us examine the dynamical behavior in the absence of perturbation. Injecting (5) into (2), we obtain

$$\hat{\kappa}_i = \kappa_{\mathsf{opt}} + (1-\mu A)^i(\hat{\kappa}_0 - \kappa_{\mathsf{opt}}). \tag{8}$$

It follows that for any initial value $\hat{\kappa}_0$ in the vicinity of κ_{opt}, the proposed algorithm has a geometric convergence to the optimal value of the scaling factor provided that the following stability condition is satisfied

$$0 < \mu < \mu_{\mathsf{max}} = \frac{2}{A}. \tag{9}$$

Conversely, assume that the transient has died out, the contribution of the perturbation to the estimation error $e_i = \hat{\kappa}_i - \kappa_{\mathsf{opt}}$ is the response of N_i to the filter whose z-transform is

$$H_N(z) = \frac{-\mu z}{z - 1 + \mu A}$$

and whose one-sided bandwidth B_L is defined by

$$2B_L T = T\int_{-1/2T}^{+1/2T} |H_N(e^{j2\pi fT})|^2 df = \frac{\mu}{A(2-\mu A)}. \tag{10}$$

It follows that the estimation error variance is obtained as

$$E[e_i^2] = T\int_{-1/2T}^{+1/2T} \sigma_N^2 |H_N(e^{j2\pi fT})|^2 df = \frac{\sigma_N^2 \mu}{A(2-\mu A)}, \tag{11}$$

since the power spectral density of the perturbation is flat according to (7).

Notice the importance of striking a balance between the rate of decay of the transient (8) and the variance of the estimation error (11), when setting the value of the step-size μ.

V. Numerical results

A. System setup

We consider a single-channel transmitter using a length-$N = 16000$ regular low-density parity-check (LDPC) [9] FEC encoder, with coding rate 0.8. The encoded data are demultiplexed to two polarization components at the symbol rate $1/T = 32$ GBaud and presented to quadrature amplitude modulators (4-QAM) before root raised-cosine pulse shaping with 10% roll-off.

The dispersion unmanaged [4] transmission line is a succession of $N_s = 20$ spans of 100 km long standard single-mode fiber (SMF). An optical amplifier yields the same launched average signal power, P_{in}, at the beginning of each span.

The SMF propagation, modeled using the split-step Fourier method [4], is characterized by the following parameters at wavelength $\lambda = 1550$ nm: an attenuation coefficient α=0.2 dB/km, a group velocity dispersion (GVD) $D = 20$ $ps.nm^{-1}.km^{-1}$, a nonlinear index $n_2 = 2.6.10^{-20}$ m^2/W, and an effective area $A_{\mathsf{eff}} = 80$ μm^2, so that the fiber nonlinearity coefficient is $\gamma = 2\pi n_2/\lambda A_{\mathsf{eff}} = 1.3174$ $km^{-1}.W^{-1}$.

At the receiver side, the coherent demodulator uses a LO with laser linewidth of 100 kHz, followed by an order-5 Bessel low-pass filter of bandwidth set to $B = 0.7/T$ and an ADC with oversampling factor 64. The noise due to amplified spontaneous emission (ASE) in optical amplifiers is then taken into account by applying an equivalent noise loading for a targeted SNR in the electrical domain. Unless otherwise specified, DSP consists of a $T/64$-spaced CD/NL-C, followed by a 21×4-tap $T/2$-spaced constant modulus algorithm (CMA) equalizer for residual CD compensation and polarization demultiplexing and a decision-directed phase-locked loop for carrier recovery [1]. CD/NL-C is implemented using either the third-order IVSTF equalizer in [3] or DBP from [4] with 10 steps per fiber span. The system performance is measured by the bit-error rate (BER) before FEC decoding, denoted by BER_{pre}. Moreover, the signal quality at the receiver is measured by the Q^2-factor, defined as $Q = 20\log_{10}(\sqrt{2}\text{erfc}^{-1}(2BER_{\mathsf{pre}}))$ (dB), where erfc(.) is the complementary error function [8].

B. Performance analysis verification

In this subsection, without loss of generality, we consider DBP-based CD/NL-C. The background noise variance is kept constant, irrespective of P_{in} and its value corresponds to an electrical signal-to-noise ratio (SNR) equal to 7.54 dB at $P_{in} = 4$ dBm. Fig. 2 plots the objective function for minimization, parameterized by the launch power P_{in}. Typically as P_{in} increases, $J(\kappa)$ first decreases thanks to improved SNR, and then increases again due to the degradation caused by nonlinearity. We observe that $J(\kappa)$ is a convex function, so that a unique minimum at κ_{opt} exists. Also, $J(\kappa)$ becomes more sensitive to a change in its argument as P_{in} increases. As a consequence, the tolerance to a deviation of the scaling factor around κ_{opt} is substantially larger close to the linear regime. Now, consider the proposed online estimation algorithm (2), using (3) to approximate the derivative of the objective function. The discretization step h should be sufficiently small to get a good approximation of the derivative. However, it was observed by Monte Carlo simulations that the variance of the perturbation term N_i in (4) grows with decreasing h. We set h to 0.05, which was found to be a good compromise. Unless otherwise specified, for the sake of fair comparison (identical computational complexity), we always fix to 20 the approximate number of codewords processed before convergence is reached. Consequently, the step-size μ is set to $\mu_{\mathsf{max}}/10$ using (9), and the corresponding numerical values are listed in Table I. The proposed algorithm is initialized to $\hat{\kappa}_0 = 0$, which corresponds to conventional linear CD compensation only (without NL compensation) [4]. As illustrated in Fig. 3, the average dynamical behavior of the proposed algorithm closely matches the theoretical behavior predicted by (8).

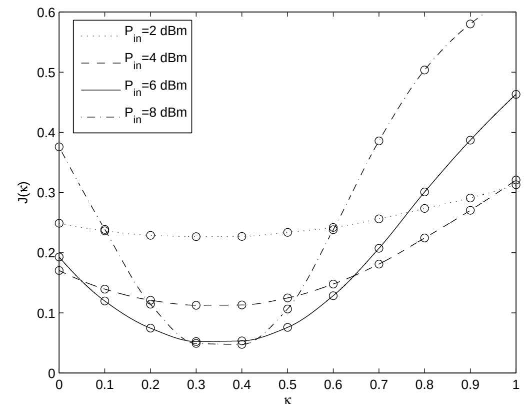

Fig. 2. Objective function plotted against the fiber nonlinearity scaling factor κ, parameterized by the signal launch power P_{in}.

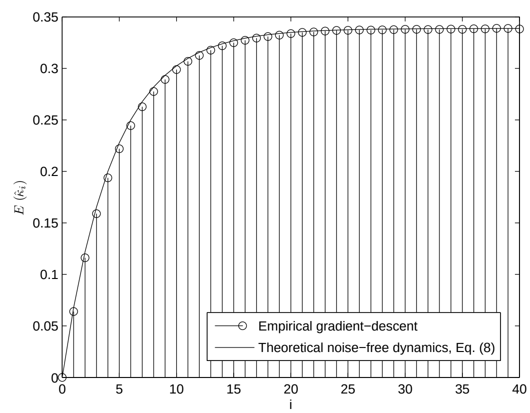

Fig. 3. Average dynamical behavior of stochastic gradient-descent at $P_{in} = 4$ dBm.

TABLE I
Step-size of the proposed algorithm applied to DBP.

P_{in}	2 dBm	4 dBm	6 dBm	8 dBm
μ	0.4958	0.1943	0.0838	0.0315

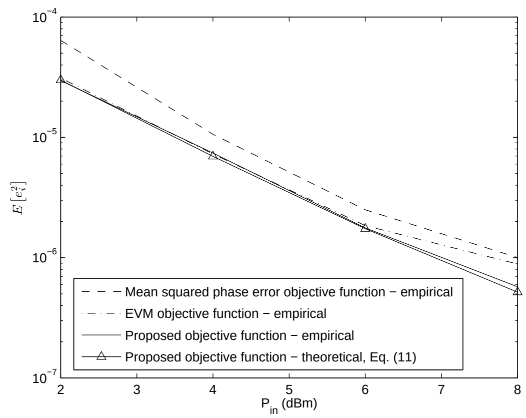

Fig. 4. Variance of the steady-state residual estimation error as a function of the signal launch power P_{in}.

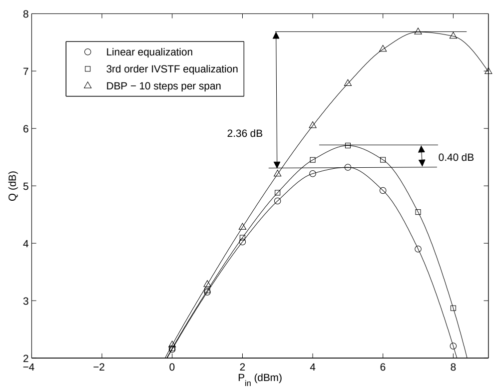

Fig. 5. Q^2-factor after the proposed nonlinear parameter correction as a function of the signal launch power P_{in}.

C. Comparison with existing methods

We now compare the objective function introduced in (1) with known metrics in nondataided reception, such as the mean squared residual phase error [7] and the error vector magnitude (EVM) [12] after carrier phase recovery. Fig. 4 shows the estimation error variance of the gradient-descent after convergence, averaged over 50 codewords. We observe that the proposed criterion outperforms the standard metrics.

Also, note that a conventional grid search [4] over Fig. 2, were each point is obtained by simulating 50 codewords to provide sufficient averaging, could reach the same accuracy as the proposed method at the expense of a complexity increase of two orders of magnitude, though.

D. Comparison of CD/NL-C algorithms

Let us now compare the performances of linear equalization [1], third-order IVSTF equalization in [3] and DBP from [4] with 10 steps per fiber span, under identical settings (i.e. electrical SNR=7.54 dB at $P_{in} = 4$ dBm, $h = 0.05$, $\mu = \mu_{\mathsf{max}}/10$ and $\hat{\kappa}_0 = 0$). Fig. 5 illustrates the Q^2-factor as a function of P_{in}, after convergence of the proposed scaling factor adjustment algorithm for the IVSTF and DBP. Unsurprisingly in the linear regime (i.e. $P_{in} \leq 2$ dBm), the three algorithms have almost the same Q^2-factor. However, in the nonlinear regime, IVSTF equalization is better than linear equalization, with a gain of 0.40 dB over standard linear equalization in terms of maximum Q^2-factor. DBP outperforms the two other methods and achieves a gain of 2.36 dB over standard linear equalization in terms of maximum Q^2-factor.

VI. Conclusion

We presented a low-complexity method to calibrate the correction factor scaling the nonlinearity parameter in DSP-based nonlinearity compensation of optical fibers. The proposed algorithm can be applied online and works in a blind context, which is useful when prior information on the fiber is either uncertain or unavailable. The calibration error has been analyzed theoretically and validated by simulations for standard methods such as IVSTF equalization and DBP. The present work is easily adaptable to higher order modulations and other variants of nonlinearity compensation. Extensions of the proposed method to more than one unknown fiber parameter will be considered in the future.

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