A message-passing receiver for OFDM-based self-interference-limited networks

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***Abstract*—In this paper, we develop a reliable bandwidth-efficient solution for coherent reception in networks prone to self-interference. Our primary focus is on OFDM-based amplify-and-forward two-way relaying, where each source node suffers from re-radiation of its own signal. An important issue is the superposition of the desired and interfering signals, which makes the channel estimation task difficult, especially for varying channel coefficients. To address this challenging problem, we introduce a factor-graph model of the estimation and detection problem at hand. Exploiting the conditional independence between unknown variables in the factor-graph, an efficient message-passing algorithm, that performs joint Bayesian channel estimation, self-interference mitigation and decoding, is derived. Simulation results show that the performance of the proposed method is close to that given by perfect knowledge of the channel for the target and interfering signals, even with limited training overhead.**

***Index Terms*—Self-interference cancellation, two-way relay networks, OFDM, joint channel estimation and detection, factor-graph, message-passing receiver.**

I. Introduction

In order to increase the network throughput, recently proposed communications systems accept that a node can be affected by a perturbation originating from itself. This perturbation, referred to as self-interference, is superposed to the signal of interest and can be the limiting factor of the system if its contribution cannot be mitigated down to the thermal noise level. Examples of emerging communication networks using this paradigm to boost the throughput include full-duplexing [1]-[3], where transceivers transmit and receive at the same time on the same frequency band, multi hop wireless protocols, where collisions with earlier received packets can be resolved [4], and analog network coding for two-way relays [5], where a destination node receives a weighted sum of the its own signal along with the intended signal. Even in one-way protocols, where self-interference is absent, joint channel estimation and detection, especially in the case of varying channel coefficients, can be a non-trivial task [6]. It follows that in the presence of self-interference, channel estimation and detection become even more complicated, since we have a multiple received overlapping signal problem. Indeed, the channel and data of the intended signal are *a priori* unknown. Also, the self-interference channel must be estimated with high accuracy and combined with the known self-interference data to provide proper self-interference cancellation.

In this paper, we focus on orthogonal frequency-division multiplexing (OFDM) modulated amplify-and-forward two-way relaying, although the proposed algorithm is relevant for all the aforementioned applications. The superposition of the signals from the source nodes at the relay antenna is broadcast using the amplify-and-forward strategy [5], which in turn generates self-interference at the source nodes. Other strategies, such as denoise-and-forward or decode-and-forward [7], are not considered here due to increased relay processing complexity.

Several self-interference cancellation methods suitable for OFDM modulated amplify-and-forward two-way relaying have been investigated in the litterature. Self-interference cancellation has been introduced in [8] under the assumption of perfect channel impulse response (CIR) or channel frequency response (CFR) knowledge. For practical environments, joint estimation of the intended and self-interference node-to-relay-to-node composite channels is mandatory. These techniques can loosely be classified as follows: blind, data-aided (DA) and iterative estimation methods. Blind composite CFR estimation based on second order statistics is investigated in [9], with the drawback that a significant number of OFDM symbols must be devoted to the estimation of the correlation matrix of the received signal. DA methods perform channel estimation based on a known OFDM block, with the implicit assumption that the channel will remain quasi-static during subsequent data blocks. For instance, the least square (LS) and the linear minimum mean square error (LMMSE) estimator are used for DA composite CIR estimation in [10] and [11], respectively. DA-LS composite CFR estimation is also investigated in [12]. In addition to known training symbols, iterative channel estimation also exploits *a priori* information on the data symbols, in order to better reconstruct the channel [13]. *A priori* information originates from soft demodulation, equalization, or channel decoding, which in turn benefits from the resulting improved channel reconstruction. In the context of two-way relaying with OFDM modulation, iterative composite CIR estimation methods using expectation conditional maximization [14] and soft-input Kalman filtering [15], have appeared.

In this paper, we introduce a frequency-domain self-interference mitigation method, suitable for block fading channels. Unlike most aforementioned methods, that require the wireless channel to be constant between two training blocks, we accomodate faster time variations by assuming the channel to be constant only over each OFDM symbol. Processing is performed in the frequency domain, so we need to estimate the composite intended and self-interference CFR with the

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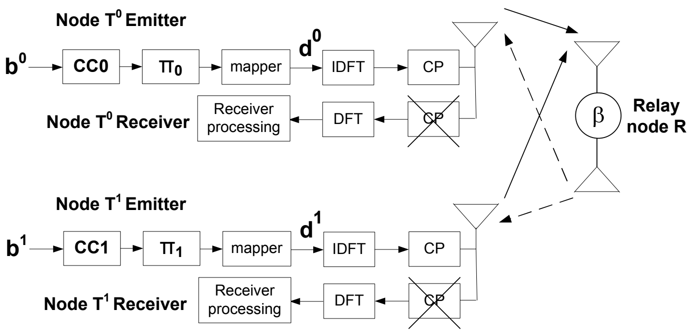

Fig. 1. OFDM-based amplify-and-forward two-way relaying: solid (resp. dashed) arrows represent the first or MAC (resp. second or broadcast) phase.

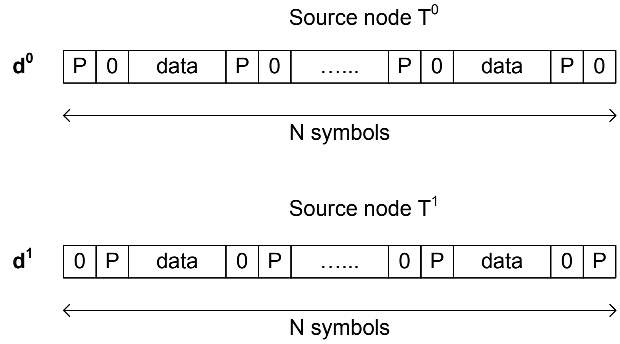

Fig. 2. Data format at the source nodes: pilot symbols (P), zero values (0) and coded data symbols (data) are assigned to the N subcarriers.

help of a limited number of pilot symbols, spread among the subcarriers. After introducing a state-space model for the CFRs and an observation model for the received baseband signal, we propose a factor-graph representation [16]-[17] of the problem at hand. As a result, we derive a semi-blind message-passing algorithm exploiting channel coding for the purpose of reliable joint channel estimation, self-interference mitigation and data detection. Since we are in the presence of a complicated mixed discrete-continuous Bayesian inference estimation problem, the calculation of continuous-valued messages requires some form of approximation, since it involves multiplications and integrals that cannot in general be expressed in closed form. To get a tractable solution, we resort to a Gaussian approximation, in the same spirit as belief propagation in Gaussian graphical models [18].

The proposed algorithm has several distinctive features with respect to methods available in the literature. First, conventional receiver processing substracts the self-interference, based on estimating the self-interference channel. Therefore, low residual self-interference requires high estimation accuracy. However unlike in our method, at the self-interference mitigation and at the demodulation level, there is generally no principled way to take the estimation accuracy into account. Secondly, exploiting the conditional independences between the hidden and observed variables, leads to separate Bayesian estimators for the intended and self-interference channels, where coupled estimation is in order in existing methods. Thirdly, we show how the improved detection performances of the proposed method enable reduced pilot overhead.

Throughout the paper, $\mathcal{N}(x : m, P)$ (resp. $\mathcal{N}_\mathcal{C}(x : m, P)$) denotes a real (resp. complex) Gaussian distribution of the variable x, with mean m and variance P.

This paper is organized as follows. First, Sec. II describes the system model for OFDM modulated amplify-and-forward two-way relaying. In Sec. III the exact message-passing algorithm for joint channel estimation, demodulation and decoding is derived. In Sec. IV, a low-complexity receiver based on Gaussian messages is introduced. Sec. V analyses the convergence behavior. Finally, simulation results in Sec. VI, compare the proposed method with existing algorithms.

II. System model

A. Communication system

We consider the bidirectional communication scheme illustrated in Fig. 1, where two terminal nodes T^0 and T^1 exchange their packets within two time slots, with the help of a relay node R. All nodes are subject to the half-duplex constraint (i.e. a node is not allowed to transmit and receive at the same time), and are equipped with a single antenna.

During the first phase (or multiple access phase), T^0 and T^1 send their packets simultaneously to R. Terminal node T^i generates a sequence of independently and uniformly distributed information bits, $\mathbf{b}^i = [b_1^i, b_2^i, \ldots, b_B^i]^T$. Using bit-interlaved coded modulation (BICM) [19], $\mathbf{b}^i$ is encoded by the convolutional code CC^i, passed trough the bit interleaver π^i and mapped into complex symbols. Also, pilot symbols are periodically inserted for the purpose of channel estimation, as depicted in Fig. 2, to obtain the modulated vector $\mathbf{d}^i = [d^i(0), d^i(1), \ldots, d^i(N-1)]^T$, with the normalization $E[|d^i(n)|^2] = 1 \ \forall n$. We use the notation $\mathbf{d}^i = \mathcal{C}^i(\mathbf{b}^i)$, where $\mathcal{C}^i$ denotes the deterministic transformation corresponding to the combined effect of channel coding, interleaving and pilot insertion. Then an OFDM symbol is obtained by feeding $\sqrt{E_s}\mathbf{d}^i$ to an N-point inverse discrete Fourier transform (IDFT) and adding a length-G cyclic prefix (CP) [1], where E_s denotes the average energy per transmitted symbol.

During the second phase (or broadcast phase), a relay with limited processing capability, amplifies the received signal by a constant factor β and forwards it to the terminal nodes [5].

B. Channel modeling and simulation

The frequency-selective fading channel between T^i and R (resp. between R and T^i) is modeled by a discrete-time CIR $\mathbf{h}_{0,i} = [h_{0,i}(0), \ldots, h_{0,i}(L-1)]^T$ (resp. $\mathbf{h}_{1,i} = [h_{1,i}(0), \ldots, h_{1,i}(L-1)]^T$), using the wide-sense stationary uncorrelated scattering (WSSUS) model [20]. Assuming quasi-static fading, for each OFDM symbol the CIR coefficients are drawn independently at random from a circularly symmetric complex Gaussian distribution

$$h_{0,i}(l) \sim \mathcal{N}_C(0, E\left[|h_{0,i}(l)|^2\right])$$
$$h_{1,i}(l) \sim \mathcal{N}_C(0, E\left[|h_{1,i}(l)|^2\right])$$

with $\sum_{l=0}^{L-1} E\left[|h_{0,i}(l)|^2\right] = \sum_{l=0}^{L-1} E\left[|h_{1,i}(l)|^2\right] = 1$. As a result, the CFR between T^i and R, $\{H_{0,i}(n)\}_{n=0}^{N-1}$ (resp. between R and T^i, $\{H_{1,i}(n)\}_{n=0}^{N-1}$) is obtained from the zero-padded N-point discrete Fourier transform (DFT) of $\mathbf{h}_{0,i}$ (resp. of $\mathbf{h}_{1,i}$). It follows that $\forall n$, $H_{0,i}(n)$ and $H_{1,i}(n)$ are random variables with prior $\mathcal{N}_C(0,1)$. Moreover, the channel noise is modeled by a zero-mean complex additive white Gaussian noise (AWGN) term with variance N_0. Here, we choose

$$\beta = \sqrt{\frac{E_s}{2E_s + N_0}}$$

in order to normalize the average transmitted energy per symbol to E_s at R.

Let us define the self-interference and intended composite CFR at T^0 on the n-th subcarrier as $X^0(n)$ and $X^1(n)$, respectively. We have

$$\begin{aligned} X^0(n) &= \beta\sqrt{E_s} H_{0,0}(n) H_{1,0}(n) \\ X^1(n) &= \beta\sqrt{E_s} H_{0,1}(n) H_{1,0}(n). \end{aligned} \tag{1}$$

$X^1(n)$ has a complex double Gaussian prior distribution [21], whose first two moments are

$$\begin{aligned} &E\left[X^1(n)\right] = 0 \\ &E\left[|X^1(n)|^2\right] = \beta^2 E_s. \end{aligned} \tag{2}$$

Assuming channel reciprocity, $H_{0,0}(n) = H_{1,0}(n)\ \forall n$, then the distribution of $X^0(n)$ is readily available from [22] and the two first two moments are

$$\begin{aligned} &E\left[X^0(n)\right] = 0 \\ &E\left[|X^0(n)|^2\right] = 2\beta^2 E_s. \end{aligned} \tag{3}$$

Otherwise, in non-reciprocal channel environments, $X^0(n)$ has a complex double Gaussian prior distribution, whose first two moments are

$$\begin{aligned} &E\left[X^0(n)\right] = 0 \\ &E\left[|X^0(n)|^2\right] = \beta^2 E_s. \end{aligned} \tag{4}$$

Using the WSSUS assumption [20], the CFRs in (1) have zero intercorrelation, but exhibits strong autocorrelation in the frequency domain within their respective correlation bandwidths. For simplicity, we will model the self-interference and intended CFRs as independent first-order Markov processes.

C. Observation model

Without loss of generality, we consider the received baseband signal at T^0 during the second phase. We assume ideal timing and frequency synchronization, and $G \geq 2L-1$ to avoid interblock interference. Moreover, the superposed signals impinging at the relay R have different timing offsets due to different propagation delays. As shown in [23], for OFDM modulation with AF relaying, a long enough CP will solve the problem of asynchronism between the terminal nodes. After CP removal and DFT, the frequency-domain received signal is a linear combination of the modulated symbols from T^0 and T^1, for all subcarriers $n = 0, \ldots, N-1$

$$Y^0(n) = X^0(n)d^0(n) + X^1(n)d^1(n) + N^0(n), \tag{5}$$

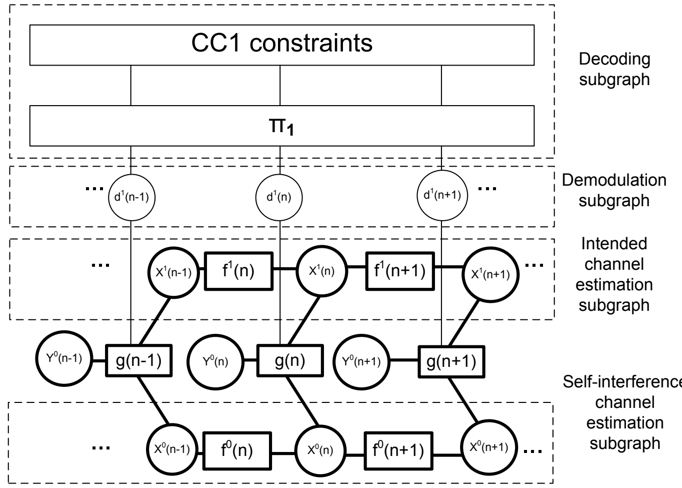

Fig. 3. Factor graph corresponding to receiver processing at source node T^0.

where the first term is the self-interference signal, the second term is the desired signal and the white noise term $N^0(n)$, having a distribution that can be approximated as $\mathcal{N}_C(0, R^0)$ where $R^0 = (\beta^2+1)N_0$, is the AWGN contribution.

D. Factor-graph framework

A factor-graph is a graphical representation of the factorization of a function of several variables [16]-[17]. Here, the function is the *a posteriori* probability density function (p.d.f.) of interest, while the variables are the unobserved state variables and the observed variables defined previously. Bayesian inference can then be performed in an efficient way by applying the sum-product algorithm (SPA) [16]-[17].

At node T^0, the self and intended CFR, the modulated data and the bit sequence sent by T^1 belong to the unobserved variables, while the bit sequence sent by T^0 and the noisy observations (5) belong to the observed variables. Therefore, the p.d.f. of interest has the form

$$\begin{aligned} &p\left(\{X^0(n)\}_{n=0}^{N-1}, \{X^1(n)\}_{n=0}^{N-1}, \mathbf{d}^1, \mathbf{b}^1 | \{Y^0(n)\}_{n=0}^{N-1}, \mathbf{d}^0\right) \\ &\propto p\left(\{Y^0(n)\}_{n=0}^{N-1}, \{X^0(n)\}_{n=0}^{N-1}, \{X^1(n)\}_{n=0}^{N-1}, \mathbf{d}^1, \mathbf{b}^1 | \mathbf{d}^0\right), \end{aligned}$$

where the second line in obtained from Bayes's rule. Now, assuming statistical independence between the self-interference CFR, the intended CFR and the transmitted data, we obtain the following factorization

$$\begin{aligned} &p\left(\{X^0(n)\}_{n=0}^{N-1}, \{X^1(n)\}_{n=0}^{N-1}, \mathbf{d}^1, \mathbf{b}^1 | \{Y^0(n)\}_{n=0}^{N-1}, \mathbf{d}^0\right) \\ &\propto p\left(\{Y^0(n)\}_{n=0}^{N-1} | \{X^0(n)\}_{n=0}^{N-1}, \{X^1(n)\}_{n=0}^{N-1}, \mathbf{d}^1, \mathbf{d}^0\right) \\ &\times p(\{X^0(n)\}_{n=0}^{N-1}) p(\{X^1(n)\}_{n=0}^{N-1}) p(\mathbf{d}^1|\mathbf{b}^1) p(\mathbf{b}^1). \end{aligned}$$

This expression can be further simplified by using the memoryless channel assumption (see Sec. II-C), the first-order Markov assumption for the self and intended CFR (see Sec. II-B), the deterministic transformation relating $\mathbf{d}^1$ to $\mathbf{b}^1$ (see Sec. II-A) and the fact that the information bits are independently and uniformly distributed (see Sec. II-A)

$$
\begin{aligned}
&p\left(\{X^0(n)\}_{n=0}^{N-1}, \{X^1(n)\}_{n=0}^{N-1}, \mathbf{d}^1, \mathbf{b}^1 | \{Y^0(n)\}_{n=0}^{N-1}, \mathbf{d}^0\right) \\
&\propto \prod_{n=0}^{N-1} p\left(Y^0(n)|X^0(n), d^0(n), X^1(n), d^1(n)\right) \\
&\quad \times p(X^0(0)) \prod_{n=1}^{N-1} p(X^0(n)|X^0(n-1)) \\
&\quad \times p(X^1(0)) \prod_{n=1}^{N-1} p(X^1(n)|X^1(n-1)) \\
&\quad \times I(\mathbf{d}^1 = \mathcal{C}^1(\mathbf{b}^1)).
\end{aligned} \tag{6}
$$

where $I(.)$ denotes the indicator function. The factor-graph corresponding to the above factorization is depicted in Fig. 3. Variable nodes are represented as circles and the local functions appearing in the factorization, denoted by

$$
\begin{aligned}
f^0(n) &= p(X^0(n)|X^0(n-1)) \\
f^1(n) &= p(X^1(n)|X^1(n-1)) \\
g(n) &= p\left(Y^0(n)|X^0(n), d^0(n), X^1(n), d^1(n)\right)
\end{aligned} \tag{7}
$$

are represented as squares. We notice that the resulting factor-graph can be decomposed into several subgraphs corresponding to distinct Bayesian estimation tasks, namely the self-interference CFR estimation subgraph, the intended CFR estimation subgraph and the demodulation and decoding subgraph.

III. Exact message-passing receiver

In this section, we derive the messages propagated by the sum-product algorithm along the edges of the factor-graph [16]-[17], without any approximation. Let $\mu_{u\to v}(.)$ denote the message sent by node u to node v in the factor-graph.

A. Self-interference CFR estimation subgraph

The incoming messages on the self-interference CFR estimation subgraph are computed from the sum-product rule applied at the factor node $g(n)$

$$
\begin{aligned}
&\mu_{g(n)\to X^0(n)}(X^0(n)) \\
&\propto \sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n)) \\
&\quad \times \int p\left(Y^0(n)|X^0(n), d^0(n), X^1(n), d^1(n)\right) \\
&\qquad \times \mu_{X^1(n)\to g(n)}(X^1(n)) dX^1(n).
\end{aligned} \tag{8}
$$

The above message acts as the conditional likelihood of observation $Y^0(n)$, averaged over the discrete symbols $d^1(n)$ sent by T^1 and the intended CFR $X^1(n)$.

Then, the forward messages on the self-interference CFR estimation subgraph are obtained from the recursion

$$
\begin{aligned}
&\mu_{f^0(n+1)\to X^0(n+1)}(X^0(n+1)) \\
&\propto \int p(X^0(n+1)|X^0(n)) \mu_{g(n)\to X^0(n)}(X^0(n)) \\
&\qquad \times \mu_{f^0(n)\to X^0(n)}(X^0(n)) dX^0(n).
\end{aligned} \tag{9}
$$

Similarly, the backward messages are obtained from the recursion

$$
\begin{aligned}
&\mu_{f^0(n)\to X^0(n-1)}(X^0(n-1)) \\
&\propto \int p(X^0(n)|X^0(n-1)) \mu_{g(n)\to X^0(n)}(X^0(n)) \\
&\qquad \times \mu_{f^0(n+1)\to X^0(n)}(X^0(n)) dX^0(n).
\end{aligned} \tag{10}
$$

Finally, the outgoing messages from the self-interference CFR estimation subgraph are computed from the sum-product rule applied at the variable node $X^0(n)$

$$
\begin{aligned}
&\mu_{X^0(n)\to g(n)}(X^0(n)) \\
&\quad \propto \mu_{f^0(n)\to X^0(n)}(X^0(n)) \mu_{f^0(n+1)\to X^0(n)}(X^0(n)).
\end{aligned} \tag{11}
$$

Note that (8) acts as the conditional likelihood of a single observation, averaged over the distribution of the intended channel and the modulated symbol sent by T^1, while (9)-(11) act as a forward-backward [16] self-interference channel estimator.

B. Intended CFR estimation subgraph

Since the self-interference and intended CFR estimation subgraph have the same tree-like structure, the messages exchanged over the intended CFR estimation subgraph have the same expression as in Sec. III-A, replacing X^0 by X^1 and f^0 by f^1.

C. Demodulation and decoding subgraph

Demodulating the desired symbols consists of computing symbol-by-symbol *a posteriori* probabilities given by $\mu_{g(n)\to d^1(n)}(d^1(n))$. Applying the sum-product rule at the function node $g(n)$,

$$
\begin{aligned}
&\mu_{g(n)\to d^1(n)}(d^1(n)) \\
&\propto \int\int p\left(Y^0(n)|X^0(n), d^0(n), X^1(n), d^1(n)\right) \\
&\qquad \times \mu_{X^0(n)\to g(n)}(X^0(n)) \\
&\qquad \times \mu_{X^1(n)\to g(n)}(X^1(n)) dX^0(n) dX^1(n),
\end{aligned} \tag{12}
$$

where the two last terms in the integrand have been obtain in Sec III-A and Sec III-B, respectively. Note that since the channel variables $X^0(n)$ and $X^1(n)$ are integrated out, the soft demodulation step takes the channel estimation uncertainty explicitly into account.

Then, the message from $d^1(n)$ to the bit interleaver π_1 and from the bit interleaver π_1 to $d^1(n)$, corresponds to the bit-level probabilities computed by the turbo-demodulation algorithm introduced in [24]. Furthermore, it is well-known that exact message-passing on the decoding subgraph is implemented by the BCJR algorithm [25].

IV. Low-complexity receiver design

The continuous-valued messages obtained in Sec. III require some form of approximation, since the involved multiplications and integrals cannot be expressed in closed form. To achieve this goal, we seek a canonical distribution [26], so that the corresponding recursions boil down to the update of a limited number of parameters. Here, we adopt complex

Gaussian messages to allow tractable derivations and thus reasonable complexity. To this end, we first model the channel CFRs as Gaussian processes. It follows that to obtain the channel estimation messages, it suffices to update the mean and variance. Our simulations will demonstrate that this approach, although not exact, is sufficient if only the first and second order moments are of interest.

A. Channel state-space model

We introduce a state-space model, which approximates the self-interference and intended channel CFR as Gaussian processes. We adopt the random walk model introduced in [27]

$$\begin{cases} X^0(n) = X^0(n-1) + \Delta u^0(n) \\ X^1(n) = X^1(n-1) + \Delta u^1(n), \end{cases} \tag{13}$$

where $\Delta u^0(n) \sim \mathcal{N}_C(0, q^0)$ and $\Delta u^1(n) \sim \mathcal{N}_C(0, q^1)$ are independent and identically distributed (i.i.d.) driving noises. The parameter q^0 (resp. q^1) must be optimized based on the coherence bandwidth [20] of the self-interference (resp. intended) composite channel CFR. An initialization in the form of a prior distribution for $X^0(0)$ and $X^1(0)$ is needed, which is chosen as the circularly symmetric complex Gaussian with parameters given by (2)-(4).

B. Self-interference CFR estimation subgraph

Assume that $\mu_{X^1(n)\to g^1(n)}(X^1(n))$ is parameterized by the Gaussian message

$$\mu_{X^1(n)\to g(n)}(X^1(n)) \propto \mathcal{N}_C(X^1(n) : X^1_{n\backslash n}, P^1_{n\backslash n}) \tag{14}$$

and the the symbol-level probabilities

$$\mu_{d^1(n)\to g(n)}(d^1(n)) \tag{15}$$

are available for all possible values of $d^1(n)$ in the modulation alphabet. According to (5), (8) becomes

$$\begin{aligned} &\mu_{g(n)\to X^0(n)}(X^0(n)) \\ &\propto \sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n)) \\ &\qquad \times \int \mathcal{N}_C(Y^0(n) : X^0(n)d^0(n) + X^1(n)d^1(n), R^0) \\ &\qquad\quad \times \mathcal{N}_C(X^1(n) : X^1_{n\backslash n}, P^1_{n\backslash n}) dX^1(n) \\ &\propto \sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n)) \times \\ &\mathcal{N}_C\left(Y^0(n) : X^0(n)d^0(n) + X^1_{n\backslash n}d^1(n), |d^1(n)|^2 P^1_{n\backslash n} + R^0\right). \end{aligned} \tag{16}$$

In order to get a tractable expression for the purpose of self-interference channel estimation, we collapse (16) to a Gaussian messsage. Using the moment-matching technique recalled in Appendix A and assuming that the normalization $\sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n)) = 1$ holds

$$\begin{aligned} &\mu_{g(n)\to X^0(n)}(X^0(n)) \\ &\propto \mathcal{N}_C\left(Y^0(n) : X^0(n)d^0(n) + b^1(n), S^1(n)\right). \end{aligned} \tag{17}$$

The bias term $b^1(n)$ is the average contribution of the desired data sent by T^1

$$b^1(n) = X^1_{n\backslash n}\tilde{d}^1(n), \quad \tilde{d}^1(n) = \sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n)) d^1(n)$$

where $\tilde{d}^1(n)$ is the soft symbol estimate on n-th subcarrier. The variance $S^1(n)$ accounts for the channel noise and the residual uncertainty associated with the intended data and channel estimates

$$\begin{aligned} S^1(n) &= R^0 \\ &+ \sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n)) \\ &\qquad \times \left\{ P^1_{n\backslash n}|d^1(n)|^2 + |d^1(n) - \tilde{d}^1(n)|^2 |X^1_{n\backslash n}|^2 \right\}. \end{aligned}$$

Now, we inject the result (17) in the recursions (9) and (10). Let the corresponding Gaussian messages be

$$\begin{aligned} &\mu_{f^0(n)\to X^0(n)}(X^0(n)) \propto \mathcal{N}_C(X^0(n) : X^0_{n|n-1}, P^0_{n|n-1}) \\ &\mu_{f^0(n+1)\to X^0(n)}(X^0(n)) \\ &\propto \mathcal{N}_C(X^0(n) : X^0_{n|n+1:N-1}, P^0_{n|n+1:N-1}), \end{aligned} \tag{18}$$

the mean and variance update rule in the first and second line boils down to the standard forward and backward Kalman prediction, respectively. Finally, injecting (18) into (11) leads to

$$\mu_{X^0(n)\to g(n)}(X^0(n)) \propto \mathcal{N}_C(X^0(n) : X^0_{n\backslash n}, P^0_{n\backslash n}), \tag{19}$$

where the smoothed self-interference CFR mean and variance are expressed as

$$\begin{aligned} P^0_{n\backslash n} &= \frac{P^0_{n|n-1} P^0_{n|n+1:N-1}}{P^0_{n|n-1} + P^0_{n|n+1:N-1}} \\ X^0_{n\backslash n} &= P^0_{n\backslash n}\left[\frac{X^0_{n|n-1}}{P^0_{n|n-1}} + \frac{X^0_{n|n+1:N-1}}{P^0_{n|n+1:N-1}}\right]. \end{aligned}$$

C. Intended CFR estimation subgraph

In a similar manner, we calculate

$$\begin{aligned} &\mu_{g(n)\to X^1(n)}(X^1(n)) \\ &\propto \sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n)) \\ &\quad \times \int p\left(Y^0(n)|X^0(n), d^0(n), X^1(n), d^1(n)\right) \\ &\qquad \times \mu_{X^0(n)\to g(n)}(X^0(n)) dX^0(n). \end{aligned} \tag{20}$$

Using (5) and (19) results in

$$\begin{aligned} &\mu_{g(n)\to X^1(n)}(X^1(n)) \\ &\propto \sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n)) \times \\ &\mathcal{N}_C\left(Y^0(n) : X^0_{n\backslash n}d^0(n) + X^1(n)d^1(n), |d^0(n)|^2 P^0_{n\backslash n} + R^0\right), \end{aligned} \tag{21}$$

which is again collapsed to a Gaussian message (see Appendix B for the demonstration)

$$\begin{aligned}&\mu_{g(n)\to X^1(n)}(X^1(n))\\ &\propto\mathcal{N}_C\left(Y^0(n):X^1(n)\tilde{d}^1(n)+b^0(n),S^0(n)\right),\end{aligned} \quad (22)$$

where the bias term $b^0(n)$ is the average self-interference

$$b^0(n)=X^0_{n\backslash n}d^0(n),$$

and the variance $S^0(n)$ accounts for the channel noise, the estimation accuracy associated with the self-interference and intended channel, as well as the residual uncertainty about the desired data

$$\begin{aligned}S^0(n)=&R^0+P^0_{n\backslash n}|d^0(n)|^2\\ &+\sum_{d^1(n)}\mu_{d^1(n)\to g(n)}(d^1(n))|d^1(n)-\tilde{d}^1(n)|^2\beta^2E_s.\end{aligned}$$

It follows that if, as per hypothesis, the intended CFR estimation messages are Gaussian of the form

$$\begin{aligned}&\mu_{f^1(n)\to X^1(n)}(X^1(n))\propto\mathcal{N}_C(X^1(n):X^1_{n|n-1},P^1_{n|n-1})\\ &\mu_{f^1(n+1)\to X^1(n)}(X^1(n))\\ &\propto\mathcal{N}_C(X^1(n):X^1_{n|n+1:N-1},P^1_{n|n+1:N-1}),\end{aligned} \quad (23)$$

the corresponding mean and variance update rule is given by a conventional forward and backward Kalman prediction. Consequently, the smoothed intended CFR estimation message is obtained as

$$\mu_{X^1(n)\to g(n)}(X^1(n))\propto\mathcal{N}_C(X^1(n):X^1_{n\backslash n},P^1_{n\backslash n}), \quad (24)$$

where

$$\begin{aligned}P^1_{n\backslash n}&=\frac{P^1_{n|n-1}P^1_{n|n+1:N-1}}{P^1_{n|n-1}+P^1_{n|n+1:N-1}}\\ X^1_{n\backslash n}&=P^1_{n\backslash n}\left[\frac{X^1_{n|n-1}}{P^1_{n|n-1}}+\frac{X^1_{n|n+1:N-1}}{P^1_{n|n+1:N-1}}\right].\end{aligned}$$

D. Demodulation and decoding subgraph

Using (19) and (24), the soft demodulation formula (12) simplifies to the computation of

$$\begin{aligned}&\mu_{g(n)\to d^1(n)}(d^1(n))\\ &\propto\mathcal{N}_C\Big(Y^0(n):X^0_{n\backslash n}d^0(n)+X^1_{n\backslash n}d^1(n),\\ &\qquad |d^0(n)|^2P^0_{n\backslash n}+|d^1(n)|^2P^1_{n\backslash n}+R^0\Big).\end{aligned} \quad (25)$$

In this expression, the first term of the mean corresponds to implicit self-interference cancellation, while the two first terms of the variance account for the self-interference and intended CFR estimation accuracy. Then, (25) serves as the observation likelihood conditioned on the symbol values $d^1(n)$, at the input of a standard turbo-demodulation algorithm, as shown in Sec. III-C.

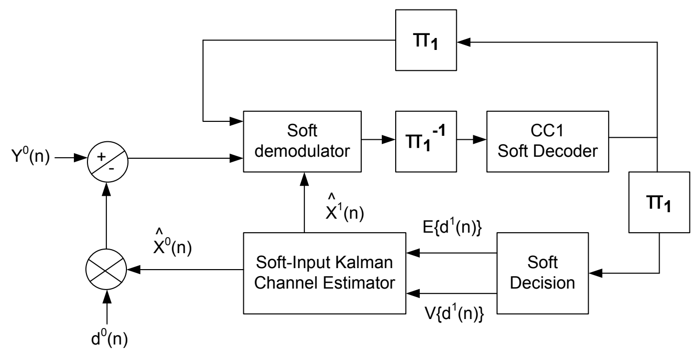

Fig. 4. Iterative self-interference cancellation and turbo-demodulation receiver with soft-input Kalman channel estimation.

E. Message-passing schedule

Since the factor-graph in Fig. 3 has cycles, the proposed low-complexity receiver has an iterative structure. Each iteration performs message-passing on the self-interference CFR estimation subgraph (see Sec. IV-B), then on the intended CFR estimation subgraph (see Sec. IV-C) and subsequently on the demodulation and decoding subgraph (see Sec. IV-D).

At the first iteration, a suitable initialization of the intended CFR message (14) and the symbol-level probabilities (15) is needed. (14) is initialized with the Gaussian prior p.d.f. by setting $X^1_{n\backslash n}=0$ and $P^1_{n\backslash n}=\beta^2E_s$. Similarly, (15) is initialized with the uniform probability mass function for data subcarriers and with a Kronecker delta function for pilot subcarriers.

F. Complexity evaluation

The computational complexity of forward or backward Kalman filtering, as well as Kalman smoothing increases as the cube of the state size [30]. Therefore, Kalman processing has complexity $\mathcal{O}(1)$ per symbol and per iteration. The complexity of the summations over all possible modulated symbols for mean/variance estimation in (17) and (22), as well as in the bitwise soft demodulation step is $\mathcal{O}(2^m)$, where m denotes the modulation order. It follows that the computational complexity of the CFR estimation and demodulation steps is $\mathcal{O}(2^m)$ per symbol and per iteration.

Besides, the computational complexity of the BCJR decoding step is $\mathcal{O}(2^{\nu+1})$ per decoded information bit and per iteration [25], where ν denotes the memory of the convolutional code.

G. Comparison with existing message-passing methods

We seek a benchmark algorithm with more or less the same computational complexity than the proposed method, taken from the class of iterative detectors mentioned in Sec. I. We adapt the iterative receiver proposed in [15] to the state-space model (see Sec. IV-A) and observation model (see Sec. II-C) at hand. The resulting iterative method, depicted in Fig. 4, is basically a turbo-demodulator relying on an intended CFR estimator $\{\hat{X}^1(n)\}_{n=0}^{N-1}$, preceded by a self-interference canceller relying on a self-interference CFR estimator $\{\hat{X}^0(n)\}_{n=0}^{N-1}$. Coupled estimation of the self-interference and intended CFR

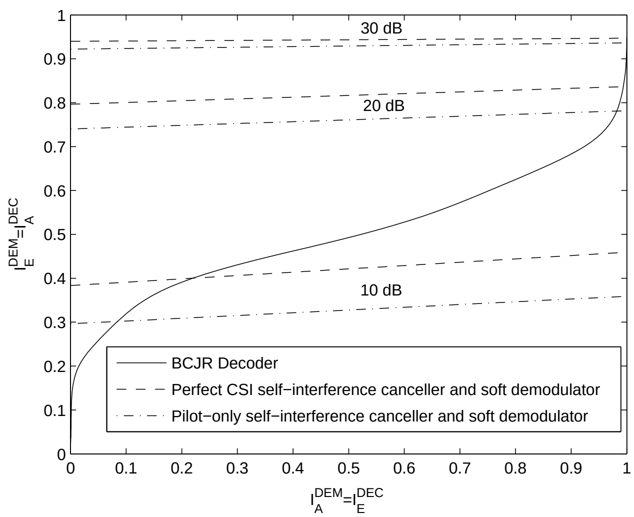

Fig. 5. EXIT chart of the proposed method for 8-QAM with perfect CSI or pilot-only channel estimation, parameterized by different values of E_s/N_0.

uses the soft-input Kalman estimator [29], fed with the expectation and variance of the desired symbols $\{d^1(n)\}_{n=0}^{N-1}$. Note that, the benchmark method uses the self-interference and intended CFR estimates as if they were the true values [15]. Thus unlike the proposed method, potentially important CFR reliability information is discarded.

V. EXIT ANALYSIS

The extrinsic information transfer (EXIT) chart is a semi-analytical tool, convenient for visualizing the convergence behavior of iterative receivers [31]. The behavior of an iterative receiver is predicted by plotting the transfer characteristics of its constituent components. Let X (resp. A) be the random variable (r.v.) corresponding to the transmitted bits of interest (resp. the *a priori* log-likelihood ratios (LLRs) on the transmitted bits). With ideal interleaving between the constituent components, the *a priori* LLRs are assumed i.i.d. distributed according to a symmetric Gaussian [31] with standard deviation σ_A

$$p_A(\zeta|X=x) = \mathcal{N}\left(A : \frac{\sigma_A^2}{2}x, \sigma_A\right).$$

It follows that the mutual information between X and A, $I(X,A)$ can be expressed as a function of σ_A as follows

$$I_A = J(\sigma_A)$$
$$J(\sigma) = 1 - \int_{-\infty}^{+\infty} \frac{e^{-(\xi-\sigma^2/2)^2/2\sigma^2}}{\sqrt{2\pi}\sigma} \log_2(1+e^{-\xi})d\xi.$$

Let E denote the r.v. corresponding to the *extrinsic* LLRs on the transmitted bits of interest, delivered by a constituent component. Using Monte Carlo simulations, the histogram method introduced in [31] is used to compute the mutual information between X and E, $I(X,E)$, that can be expressed as a function of I_A and and possibly also of the signal-to-noise ratio E_s/N_0.

For the sake of tractability, the EXIT analysis is performed on the proposed method for two extreme cases

1) Perfect channel state information (CSI) case: we choose the proposed method with perfect channel knowledge, which boils down to a turbo-demodulator preceded by an interference canceller.
2) DA case: we choose the proposed method with channel estimation message-passing enabled only at the first iteration, which boils down to turbo-demodulator with pilot-only channel estimation.

A full EXIT analysis taking channel estimation message-passing into account would be much more involved, since channel estimation at a given iteration cannot be seperated from the channel accurracy obtained at the previous iteration (see (16) for instance). However, this restriction becomes almost irrelevant in most situations, where the proposed method with channel estimation performs closely to the perfect CSI case, as shown in Sec. VI. Then, we choose our two constituent components as

1) the message computation from $d^1(n)$ to the bit interleaver π_1 in the factor graph, which corresponds to the combination of self-interference cancellation and soft demodulation. The corresponding transfer characteristic is written as
$$I_E^{DEM} = T^{DEM}(I_A^{DEM}, E_s/N_0)$$
2) the message computation from the bit interleaver π_1 to $d^1(n)$, which corresponds to a BCJR decoder. The corresponding transfer characteristic is written as
$$I_E^{DEC} = T^{DEC}(I_A^{DEC}).$$

Let l denote the message-passing iteration. At $l = 0$, the mutual information at the decoder output is initialized to $I_E^{DEC}(l) = 0$ (no prior information). Subsequently, the evolution of the mutual information is described by the recursion

$$I_E^{DEC}(l) = T^{DEC}(T^{DEM}(I_E^{DEC}(l-1), E_s/N_0)). \quad (26)$$

Fig 5 illustrates the EXIT chart for the case of 8-QAM modulation [32], $N = 2048$ subcarriers, pilot insertion rate (PIR) equal to $2:6$ and rate-$1/2$ recursive systematic convolutional encoding, with generator polynomials $(1, 5/7)$ in octal. The trajectories of the proposed iterative receivers (26), are expected to converge within $2-3$ iterations towards the unique intersection point of the two transfer characteristics. As expected, in the DA case, a lower mutual information at the decoder output is reached at convergence than in the perfect CSI case. At $E_s/N_0 = 30$ dB, in the DA case (resp. the perfect CSI case) the predicted bit-error rate (BER) at convergence is 4×10^{-6} (resp. 10^{-6}), which is only slightly lower than the Monte Carlo simulated BER. The small difference is due to the assumption of i.i.d. LLRs made by EXIT analysis, which is theoretically correct only under infinite-length random bit-interleaving. This shows that there is still room for performance improvement using the proposed low-complexity receiver. Indeed, the proposed method achieves better results than its DA counterpart, mainly because it can better reconstruct the CFRs by iteratively exploiting soft-decisions from the decoder.

VI. NUMERICAL RESULTS

Both source nodes emitters implement BICM using rate-$1/2$ recursive systematic convolutional encoding, with generator polynomials $(1, 5/7)$ in octal and interleavers drawn independently at random. Modulation is performed using either binary phase shift keying (BPSK), quadrature phase shift keying (QPSK), or 8-ary quadrature amplitude modulation (8-QAM). All individual CIRs have length $L = N/8$ and mimick a rich scattering multi-path environment with quasi-static fading, using the WSSUS modeling and simulation method described in Sec. II-B. Therefore for each OFDM symbol, the discrete-time channel coefficients $h_{j,i}(l)$, $i, j = 0, 1$, $l = 0, \dots, L-1$ are drawn independently at random from $\mathcal{N}_C(0, PDP(l))$, where the power delay profile (PDP) is chosen as

$$PDP(l) = \frac{1 - e^{-3}}{1 - e^{-3L}} e^{-3l}.$$

Accordingly, the parameters of the random walk model for the composite CFRs in Sec. IV-A are optimized to $q^0 = q^1 = 0.4$.

At node T^0, the performance criterion is chosen as the BER after three iterations (since none of the considered receivers showed any improvement by further increasing the number of iterations). The proposed method with channel estimation message-passing and PIR of $2:24$ is compared to

- the one-way relaying bound with PIR equal to $2:24$, which is the proposed method in the absence of self-interference
- the proposed method with pilot-only channel estimation, which corresponds to channel estimation message-passing beeing enabled only during the first iteration (i.e. the DA case of Sec. V). In the DA case the PIR is $2:6$.
- the benchmark iterative receiver of Sec. IV-G using the same PIR as the proposed method, that is $2:24$ unless otherwise specified.

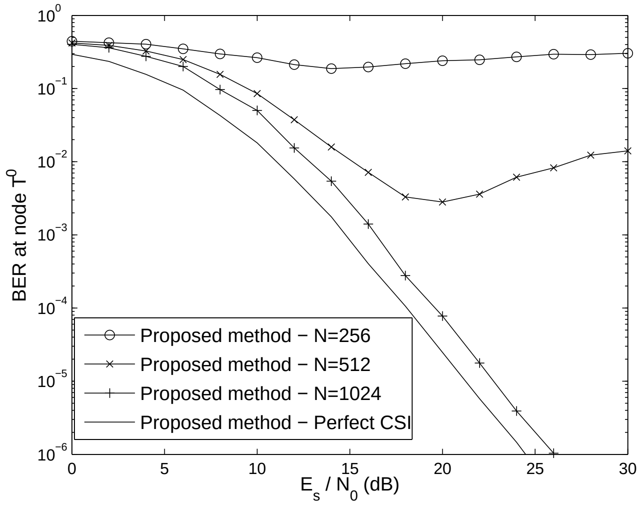

Fig. 6. BER using BPSK after 3 iterations, for the proposed method with channel estimation or perfect CSI, parameterized by different values of N.

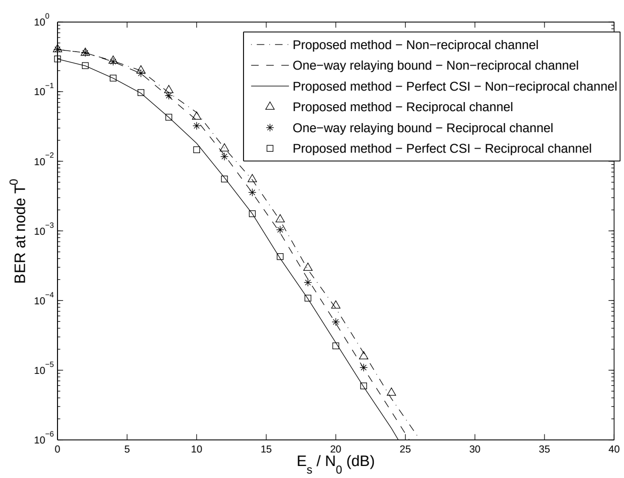

Fig. 7. BER using BPSK and $N = 1024$ after 3 iterations, for the proposed method with channel estimation or perfect CSI, in reciprocal and non-reciprocal environments. The one-way relaying bound is also shown for reference.

A. *Influence of the interleaver length*

Fig. 6 compares the BER performances at node T^0 for the BPSK modulated intended data, using the proposed message-passing receiver with channel estimation or with perfect CSI. The curves are obtained for different values of subcarrier number $N = 256, 512, 1024$. For the BER in the perfect CSI case, no dependency on N shows up. On the contrary, the BER with channel estimation is highly dependent on N. Indeed, the BER first exhibits a decreasing error floor with increasing values of N, until the BER curve finally reaches the same slope as in the perfect CSI case for $N \geq 1024$. Similar conclusions can be drawn for higher order modulations. Namely, the BER curve with channel estimation reaches the same slope as in the perfect CSI case for $N \geq 1024$ (resp. $N \geq 2048$) for QPSK (resp. 8-QAM). We interpret this phenomenon by the influence of short cycles in the decoding subgraph of the factor graph, when the interleaver size (or equivalently N) is too small.

B. *CFR Gaussian process assumption*

As indicated in Sec. II-B, the true p.d.f. of the intended CFR is complex double Gaussian. Furthermore, the true p.d.f. of the self-interference CFR is complex double Gaussian whithout channel reciprocity and given in [22] for environments with channel reciprocity. Consequently, we would like to check the applicability of the Gaussian process assumption on the composite CFRs, introduced in Sec. IV-A in order to derive our low-complexity receiver. Fig. 7 shows the BER performances of the proposed method at node T^0 for BPSK and $N = 1024$ subcarriers, are not only quasi-identical in reciprocal and non-reciprocal channel environments, but also very close to the perfect CSI case and the one-way relaying bound. In general, we found that for BPSK, QPSK and 8-QAM, as long as the BER curves with and without perfect CSI have the same slope, the power efficiency loss of the proposed method with channel estimation is only $1.5 - 2$ dB (resp. $0.65 - 1.3$ dB) with respect to the perfect CSI case (resp. the one-way relaying bound) at a BER of 10^{-5}. Thus, modeling the self-interference

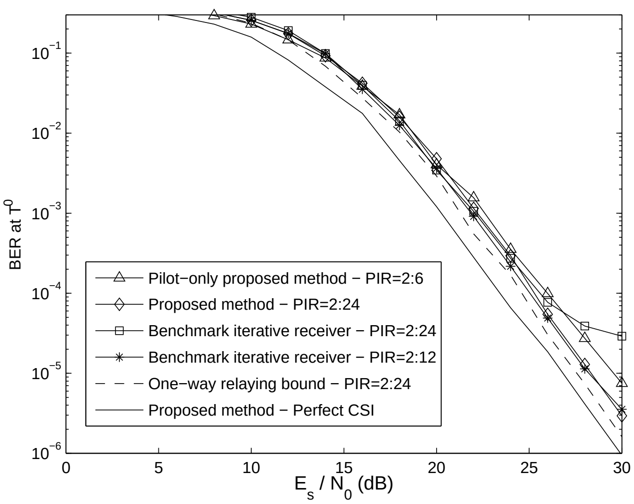

Fig. 8. BER using QPSK and $N = 1024$ after 3 iterations. The proposed method with message-passing is compared to the proposed method with pilot-only channel estimation and the benchmark iterative receiver.

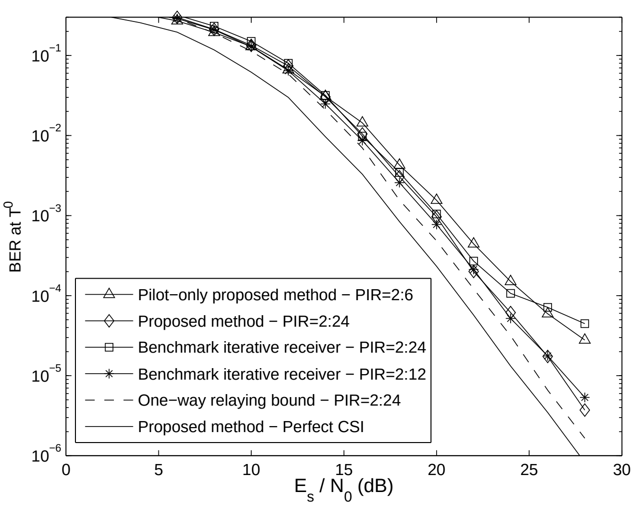

Fig. 9. BER using 8-QAM and $N = 2048$ after 3 iterations. The proposed method with message-passing is compared to the proposed method with pilot-only channel estimation and the benchmark iterative receiver.

and intended CFRs as Gaussian processes does not incur a significant performance loss, which in retrospect was to be expected since in the proposed method, CFR estimation only takes first and second order moments into account.

C. Comparison with other methods

Fig. 8 compares several iterative receivers for QPSK and $N = 1024$ subcarriers. Observe that the pilot-only method is several dBs away from the proposed method in terms of power efficiency, even when the PIR is increased by a factor of 4. The reason is that, due to rapid frequency variations, the self-interference and intended CFRs cannot be reconstructed sufficiently well, so that iterative demodulation and decoding performs as efficiently as in the proposed algorithm. Now, regarding the benchmark iterative algorithm, its is found to perform as well as the proposed scheme, but only at the price of doubling the PIR. Indeed, the disadvantage of the benchmark method is that it lacks the possibility of exploiting CFR reliability information, which in turn requires more accurate channel estimates for the purpose of self-interference cancellation and demodulation, leading to a higher PIR. Similar conclusions can be drawn for 8-QAM modulation, as illustrated in Fig. 9.

VII. Conclusion

In this paper, we introduced a factor-graph approach to joint channel estimation, interference cancellation and decoding suitable for OFDM-based self-interference limited transmissions. As a useful application, we considered bidirectional communications on a two-way relay network, over a frequency-selective block fading channel.

The proposed method relies on suitable approximations of the frequency response of the self-interference and the intended channel as Gaussian processes, in order to obtain a message-passing receiver with reasonable computational complexity.

Numerical simulations showed that, unless the pilot overhead is significantly increased, the resulting iterative receiver significantly outperforms the conventional turbo receiver with iterative channel estimation, as well as the standard pilot-only approach. Thus the proposed method's performance versus bandwidth efficiency tradeoff is advantageous in order to support reliable communications.

Future extensions of this work include the consideration of robust frequency and timing synchronization schemes, multiple-input multiple-output channels, as well as other applications like concurrent communications over wireless networks.

Appendix A
Moment-matching method

Assume that a message sent by node u to node v in the factor-graph is a Gaussian mixture of the form

$$\mu_{u\to v}(.) \propto \sum_i \omega_i \mathcal{N}_{\mathcal{C}}(., a_i, \Sigma_i), \tag{27}$$

with $\sum_i \omega_i = 1$. This message can be approximated by a single Gaussian with the same expectation and covariance as the original message, namely

$$\mu_{u\to v}(.) \propto \mathcal{N}_{\mathcal{C}}(., \hat{a}, \hat{\Sigma}), \tag{28}$$

where

$$\hat{a} = \sum_i \omega_i a_i$$
$$\hat{\Sigma} = \sum_i \omega_i \left[\Sigma_i + (a_i - \hat{a})(a_i - \hat{a})^*\right].$$

The demonstration is readily available from [28] (p. 107).

APPENDIX B

Assume the normalization $\sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n)) = 1$ holds, the moment matching method approximates (21) by $\mathcal{N}_C(Y^0(n) : m(X^1(n)), S(X^1(n)))$. The expression of the mean is

$$\begin{aligned} &m(X^1(n)) \\ &\quad = \sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n)) \left[X^0_{n\setminus n} d^0(n) + X^1(n) d^1(n)\right] \\ &\quad = X^0_{n\setminus n} d^0(n) + X^1(n)\tilde{d}^1(n), \end{aligned}$$

which is the desired result. The expression of the variance is

$$\begin{aligned} S(X^1(n)) &= R^0 + P^0_{n\setminus n}|d^0(n)|^2 \\ &+ \sum_{d^1(n)} \mu_{d^1(n)\to g(n)}(d^1(n))|d^1(n) - \tilde{d}^1(n)|^2|X^1(n)|^2. \end{aligned}$$

Now, averaging out $X^1(n)$ using (2), in the expression of the variance leads to the desired value, $S^0(n)$.

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