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Evidential Correlated Gaussian Mixture Markov Model for Pixel Labeling Problem

Lin An, Ming Li, Mohamed El Yazid Boudaren, and Wojciech Pieczynski

Abstract. Hidden Markov Fields (HMF) have been widely used in various problems of image processing. In such models, the hidden process of interest X is assumed to be a Markov field that must be estimated from an observable process Y. Classic HMFs have been recently extended to a very general model called "evidential pairwise Markov field" (EPMF). Extending its recent particular case able to deal with non-Gaussian noise, we propose an original variant able to deal with non-Gaussian and correlated noise. Experiments conducted on simulated and real data show the interest of the new approach in an unsupervised context.

Keywords: Markov random field, correlated noise model, Gaussian mixture, belief functions, theory of evidence, image segmentation

1 Introduction

The paper deals with statistical image segmentation. The use of hidden Markov fields (HMFs) has become popular since the introduction of these models in pioneering papers [1, 2] with related optimal Bayesian processing. HMFs provide remarkable results in numerous situations and continue to be used nowadays. On the other hand, Dempster-Shafer theory of evidence (DST) has been used in different information fusion problems [3, 4]. However, simultaneous use of both HMFs and DST is rather rare, and is mainly applied to fuse sensors of different nature [5, 6, 7, 8]. Another application

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Lin An, Ming Li
Xidian University, National Lab. of Radar Signal Processing;
Collaborative Innovation Center of Information Sensing and Understanding, China
email: lin.an4579@gmail.com, liming@xidian.edu.cn

Mohamed El Yazid Boudaren
Ecole Militaire Polytechnique, Computer Science, Algeria
email : boudaren@gmail.com

Wojciech Pieczynski
Telecom SudParis, CITI, France
email : wojciech.pieczynski@telecom-sudparis.eu

consists of using DST to model images with fine details, and the first results presented in [9] were encouraging. Calculations presented in [9] were possible because of the fact that DS fusion in Markov field context can be interpreted as calculation of a marginal distribution in a "triplet Markov field" (TMF [10]). The model proposed in [9] has been recently extended to non-Gaussian noise in [11], enjoying the generality of the proposed "Evidential Pairwise Markov Field" (EPMF) models. Such extensions are particularly useful in radar images context, in which noise is not Gaussian in general. The aim of this paper is to propose a further extension of the model proposed in [11] to the case of correlated noise. This seems to be of interest in radar images processing, as noises are correlated in real situations while they are usually considered independent in different Markov fields based models.

Let S be a finite set, with $Card(S) = N$, let $Y = \{Y_s\}_{s\in S}$ be the observed random field with each Y_s taking its value in $\Re$, and let $X = \{X_s\}_{s\in S}$ be the hidden random label field with each X_s taking its values from a finite set of "classes" or "labels". Realization of such random fields will be denoted using lowercase letters. The labeling problem consists in estimating $X = x$ from $Y = y$.

The reminder of the paper is organized as follows. Section 2 summarizes the theory of evidence and its applicability within Markov models. In Section 3, we describe our proposed model. In Section 4, we assess the proposed model on image segmentation. Finally, concluding remarks are presented in Section 5.

2 Background

In this section, we briefly recall the basics of Dempster-Shafer theory of evidence and discuss its application within Markov field models.

2.1 Hidden Markov fields

In basic hidden Markov fields (HMFs) context, the field X is assumed Markovian with respect to a system of cliques C, associated to some neighborhood system. The model name "hidden Markov field" stands for the very fact that the hidden field X is Markov. According to the Hammersley-Clifford equivalence, X is then an MRF given by

$$p(x) \propto \exp\left[-\sum_{c\in C}\psi_c(x_c)\right] \tag{1}$$

where $\psi_c(x_c)$ is the potential function associated to clique c, and $x_c = (x_s)_{s\in c}$.

On the other hand, the likelihood distribution $p(y|x)$ is defined by

$$p(y|x) \propto \exp\left[\sum_{s\in S}\log\left(p(y_s|x_s)\right)\right] \tag{2}$$

The joint distribution of (X,Y) is then given by

$$p(x,y) = p(x)\,p(y|x). \tag{3}$$

2.2 Theory of evidence

The class set from which X_s takes its value is defined by $L=\{l_1,\cdots l_K\}$ that is an universe of discourse, also called frame of discernment. Let $\Theta=2^L=\{\varnothing,l^1,\cdots,L\}$ $=\{\varnothing,\theta_1,\cdots,\theta_Q\}$ be its corresponding powerset, where $l^k=\{l_k\}$ and $Q=2^K-1$. A "basic belief assignment" (*bba*) is a function M from Θ to $[0,1]$ satisfying $M(\varnothing)=0$ and $\sum_{i=1}^{Q}m(\theta_i)=1$. A *bba* M defines then a "plausibility" function *Pl* and a "credibility" function *Cr*, both defined from Θ to $[0,1]$ by $Pl(\theta)=\sum_{\theta\cap\theta'\neq\phi}M(\theta')$ and $Cr(\theta)=\sum_{\theta'\subset\theta}M(\theta')$ respectively. For a given *bba* M, related *Pl* and *Cr* are linked by $Pl(\theta)+Cr(\bar{\theta})=1$. From this point of view, a probability p can be perceived as a special case for which $p=Pl=Cr$. Moreover, if two *bbas* M_1 and M_2 represent two pieces of evidence, we can merge, or fuse, them using the so called "Dempster-Shafer fusion" (DS fusion), which defines $M=M_1\oplus M_2$ given by: $M(\theta)=(M_1\oplus M_2)(\theta)$ $\propto\sum_{\theta'\cap\theta''=\theta}M_1(\theta')M_2(\theta'')$ for any $\theta\neq\varnothing$. Finally, a *bba* is said "probabilistic" or "Bayesian" when it vanishes outside singletons, and it is said "evidential" otherwise. In this paper, a probabilistic *bba* will be said defined on L and singletons l^k and elements l_k will be handled indifferently.

2.3 Hidden evidential Markov field with Gaussian-mixture likelihood

Let us consider the fields $X=(X_s)_{s\in S}$, $Y=(Y_s)_{s\in S}$ and let $p_1(x)\propto\exp\left[-\sum_{c\in C}\psi_c(x_c)\right]$ and $p^y(x)\propto\prod_{s\in S}p(y_s\mid x_s)$. p_1 and p^y will be called "prior" and "likelihood" *bbas* respectively. Then, the posterior distribution $p(x\mid y)$ given by (3) is itself the DS fusion of p_1 and p^y: $p(x\mid y)=(p_1\oplus p^y)(x)$. This is of particular significance since it may offer different possibilities of extensions [9]. More precisely, if either p_1 or p^y is extended to an evidential *bba*, the result of the fusion $p_1\oplus p^y$ remains a probabilistic distribution, which can then be seen as an extension of the classic posterior probability $p(x\mid y)$. Additionally, if the "evidential" extension of p_1 or p^y is of a similar Markovian form, the computation of posterior margins $p(x_s\mid y)$ remains feasible in spite of the fact that the fusion result is no longer necessarily a Markov field [9].

For instance, if p_1 is extended to a Markov *bba* M, we can construct an evidential Markov field (EMF) defined on Θ^N by

$$M(m) \propto \exp\left[-\sum_{c\in C}\psi_c(m_c)\right] \tag{4}$$

In [11], we consider a general situation where the priors are evidential and the noise is blind but not Gaussian. By introducing an auxiliary field $U=(U_s)_{s\in S}$ with $U_s \in \Lambda = \{\lambda_1,\cdots,\lambda_P\}$, the evidential blind Gaussian mixture Markov (EBGMM) model is given by

$$p(m,x,u,y) = 1_{x\in m}\gamma \exp\left[-\sum_{c\in C}\psi_c(m_c) - \sum_{s\in S}\eta_s(x_s,u_s) + \sum_{s\in S}Log\left(p(y_s|x_s,u_s)\right)\right] \tag{5}$$

Since $p(m,x,y) = \sum_{u\in\Lambda^N} p(m,x,u,y)$, we have

$$p(m,x,y) = \gamma\left[1_{x\in m}\exp\left[-\sum_{c\in C}\psi_c(m_c)\right]\right]\prod_{s\in S}\left[\sum_{u_s\in\Lambda}\exp[-\eta_s(x_s,u_s)]p(y_s|x_s,u_s)\right] \tag{6}$$

and thus $p(m,x,y)$ is a classic EHMF with $p(y_S|x_S)$ being mixtures, $p(y_s|x_s) = \sum_{u_s\in\Lambda}\alpha(u_s)p(y_s|x_s,u_s)$, where the mixture coefficients are $\alpha(u_s) = \exp[-\eta_s(x_s,u_s)]$. As demonstrated in [11], the interest of such models is to make it possible to deal with unknown noise densities $p(y_S|x_S)$.

3 Evidential Correlated Gaussian Mixture Markov Model

The aim of the present paper is to extend the model (5) in such a way that the possible noise correlations can be taken into account. Thus we propose a model in which the noise is non-Gaussian and correlated, and in which all parameters can be estimated by the "iterative conditional estimation" (ICE) method, allowing unsupervised image segmentation.

The distribution of the proposed model, called "evidential correlated Gaussian mixture Markov" (ECGMM) model, is written as

$$p(m,x,u,y) = 1_{x\in m}\gamma\exp\left[-\sum_{c\in C}\psi_c(m_c) - \sum_{c\in C}\phi_c(u_c) - \sum_{s\in S}\eta_s(x_s,u_s) + \sum_{s\in S}Log(p(y_s|x_s,u_s))\right] \tag{7}$$

Then the likelihood is

$$p(y|x) \propto \sum_{u\in\Lambda^N}\exp\left[-\sum_{c\in C}\phi_c(u_c) - \sum_{s\in S}\eta_s(x_s,u_s) + \sum_{s\in S}Log(p(y_s|x_s,u_s))\right] \tag{8}$$

Let us notice that this likelihood, which is new with respect to the likelihood in (5), is very different from the latter. Indeed, the likelihood in (5) verifies two classical properties:

(i) $p(y|x) = \prod_{s \in S} p(y_s|x)$;

(ii) $p(y_s|x) = p(y_s|x_s)$ for each $s \in S$,

whereas the likelihood (8) does not verify any of them. Thus the greater complexity of (7) with respect to (5) goes beyond the introduction of the noise correlation.

We have to mention that another way to construct the correlated likelihood is assuming the likelihood to be the Markov field:

$$p(y \mid x) = \gamma \exp\left[-\sum_{c \in C} \psi_c \left(y_c, x_c \right) \right], \tag{9}$$

which captures the contextual information directly [13]. Since the observation y_s takes the value from R, it is such a complex model with so many parameters. When the likelihood is simple Gaussian there are six parameters, it will be much more when we consider the Gaussian mixture. In CGMM, u_s takes the value from a limited data set, so $\psi_c(u_c)$ can be constructed by the well-used Multi-level logistic (MLL) model [14], which keeps the likelihood to be correlated as well as simplify the complexity of the model.

The labeling problem is to find $\hat{x}$ from $Y = y$. Then setting $V = (V_S)_{s \in S}$ with $V_S = (M_S, X_S, U_S)$, we have a standard hidden Markov field (V, Y). The field V is discrete finite, and thus we use the classic "iterated conditional modes" (ICM) algorithm [1, 6], which is an approximation of the optimal Bayesian solution $\hat{v}_B = \arg\max_v p(v|y)$. Having $\hat{v} = (\hat{m}, \hat{x}, \hat{u})$ gives then $\hat{x}$ (in addition, it also gives $(\hat{m}, \hat{u})$, which can be of interest). Let us consider the simplest situation: x_s takes the value from $\{l_1, l_2\}$, and u_s takes the value from $\{\lambda_1, \lambda_2\}$. Then v_s takes the value from $\Omega = \{(l_1, \lambda_1), (l_1, \lambda_2), (l_2, \lambda_1), (l_2, \lambda_2)\} = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. We can estimate the probability $p(v_s \mid y)$ on Ω by Gibbs sampler. The estimation obtained in this way enables us to compute $p(x_s = l_1 \mid y) = p(v_s = \omega_1 \mid y) + p(v_s = \omega_2 \mid y)$ and $p(x_s = l_2 \mid y) = p(v_s = \omega_3 \mid y) + p(v_s = \omega_4 \mid y)$, which are then used to perform ICM.

4 Experiments

4.1 Simulated data

The proposed model will be assessed against the existing EBGMM and HMF models on unsupervised segmentation of simulated images in both cases of independent and correlated noise. Let us consider the simulated images "*Nazca bird*", which has already been dealt with in [9, 11], and which is too complex for the simple HMFs models. There are two classes, i.e X_S takes its value from $L=\{l_1,l_2\}$, M_S takes its values from $\Theta=\{\theta_1,\theta_2,\theta_3\}=\{\{l_1\},\{l_2\},\{l_1,l_2\}\}$, and U_S takes its values from $\Lambda=\{\lambda_1,\lambda_2\}$. The non-Gaussian noise used here is the Gamma one. In independent noise case, the two noise densities are Gamma $G_1(0.5,2)$ and $G_2(3,1)$, which are quite different from Gaussian densities. The correlated noise is obtained by the following equation:

$$y_{i,j}=\left[\frac{\left(y^1_{i,j}-\mu^1\right)+\left(y^1_{i-1,j}-\mu^1\right)+\left(y^1_{i,j-1}-\mu^1\right)}{\sqrt{3}}+\mu^1\right]1_{a_{i,j}=\omega_1} +\left[\frac{\left(y^2_{i,j}-\mu^2\right)+\left(y^2_{i-1,j}-\mu^2\right)+\left(y^2_{i,j-1}-\mu^2\right)}{\sqrt{3}}+\mu^2\right]1_{a_{i,j}=\omega_2} \quad (10)$$

where i,j is the location of the pixel; y^1 and y^2 are two independent noises with the densities being $G_1(0.5,2)$ and $G_2(3,1)$; μ^1 and μ^2 are the means; α is the class image. We obtain a correlation coefficient of 0.23. We show the class image, the observed images, and their corresponding histograms in Fig. 1.

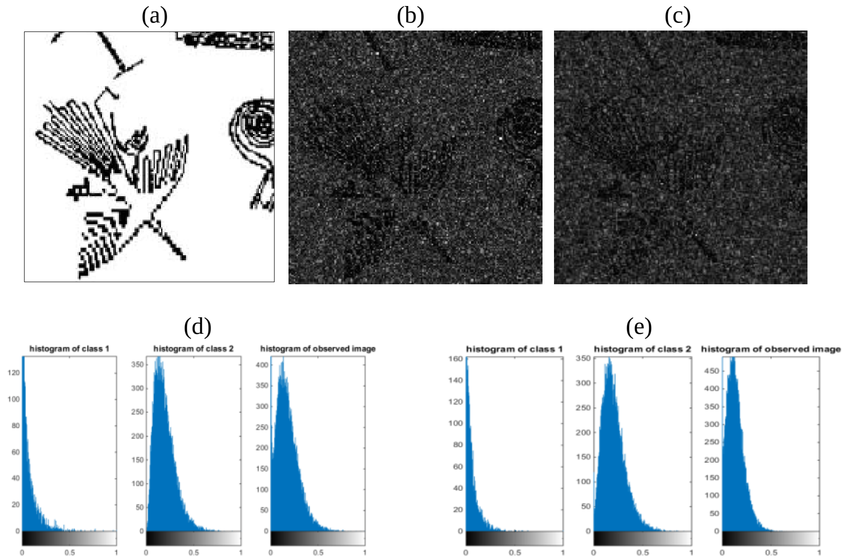

Fig. 1. Simulated noisy *Nazca bird* images (a) class image; (b) image corrupted by independent noise; (c) image corrupted by correlated noise; (d) histogram of independent noise; (e) histogram of correlated noise.

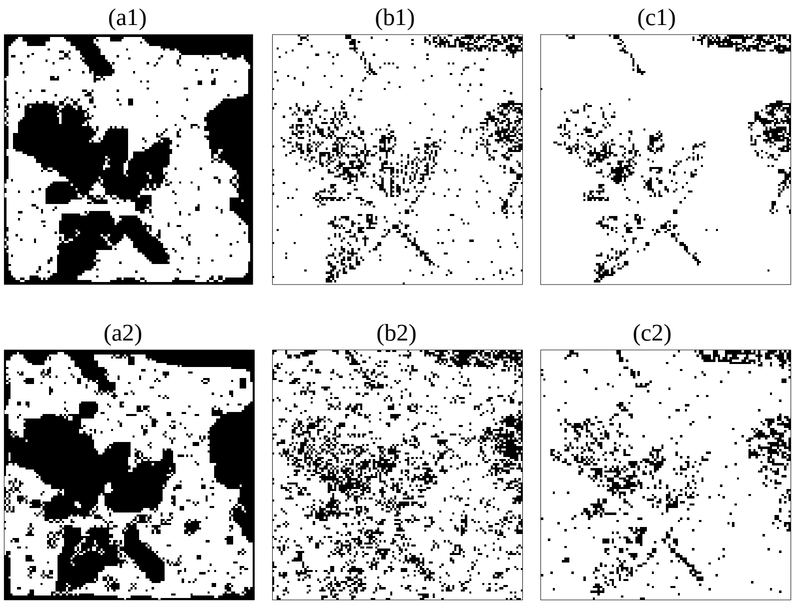

Fig. 2. Results of segmentation of noisy *Nazca bird* images. (a1-a2) by HMF; (b1-b2) by EBGMM; (c1-c2) by ECGMM. (a1-c1) independent noise case; (a2-c2) correlated noise case.

The noisy images are then segmented using HMF, EBGMM and ECGMM respectively. The obtained results are shown in Fig. 2. More precisely, we assess all approaches with respect to the reference map in terms of overall accuracy (OA) and Kappa coefficient (Kappa) [12] and illustrate them in Table 1. The best approach is the one exhibiting the highest OA, and the highest Kappa. The presented results, and other similar results obtained in additional experiments, show that HMFs give very poor results in both independent and correlated noise cases. EBGMM and ECGMM significantly improve HMFs' results in the independent-noise case, and produce equivalent results. Finally, the new ECGMM model based segmentation allows a significant improvement of the EBGMM based one in the case of correlated noise.

Table 1. Performance evaluation of different approaches on simulated images

OA (%)			
	HMF	EBGMM	ECGMM
Independent noise	73.92	**91.73**	90.28
Correlated noise	69.15	80.22	**90.61**
Kappa			
	HMF	EBGMM	ECGMM
Independent noise	0.3912	**0.5864**	0.5377
Correlated noise	0.3336	0.3405	**0.5864**

4.2 Real data

In this subsection, we evaluate our method on a real radar image. To this end, we consider the image of Toronto city, shown in Fig. 3 (a), obtained in December 2007 by TerraSAR-X SpotLight, which is single HH polarization with a resolution of $1m$.

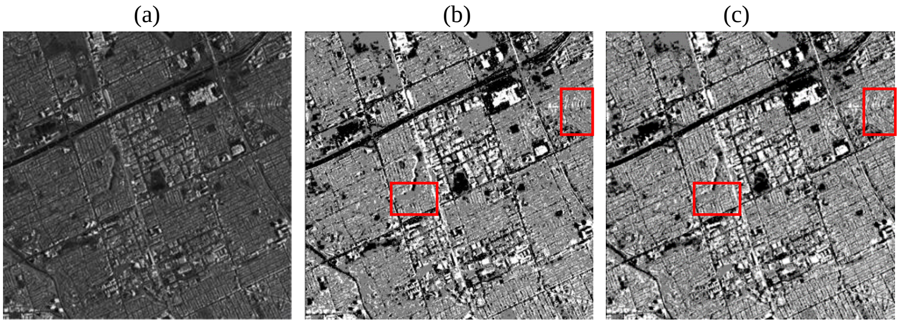

Fig. 3. Unsupervised segmentation of a real SAR image (a) real data, (b) EBGMM's result, and (c) ECGMM's result.

We segment the image into three classes by EBGMM and the proposed ECGMM, and show the result in Fig. 3. This data is full of small edges, which is a real challenge for Markov-based methods. Compared with EBGMM, we see that ECGMM seems to perform better in some spots; in particular around the rich-edge area. We can see from the red panels that the segmentation obtained by ECGMM includes more details with respect to the one obtained through EBGMM. The correlated coefficient of this data is about 0.25, which is very close to the simulated image above.

5 Conclusion

In this paper, we extended the particular "evidential pairwise Markov fields" model used in [11] to deal with the segmentation of SAR images containing fine details and non-Gaussian noise. The extension consists of introducing an auxiliary field, making it possible to take the noise correlation into account. The experiments conducted on simulated and real data prove that the new approach can significantly improve the results obtained by the previous one. In future work, one can view an extension of the probabilistic likelihood used here to an evidential one, so that the possible non stationarity of the noise could be taken into account.

References

1. Besag, J.: On the Statistical Analysis of Dirty Pictures. Journal of the Royal Statistical Society. Series B, 48(3), 259-302 (1986)

2. Geman, S., Geman, D.: Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images, IEEE Trans. on Pattern Analysis and Machine Intelligence. 6(6), 721-741 (1984)
3. Smets, P.: Belief functions: the disjunctive rule of combination and the generalized Bayesian theorem. International Journal of Approximate Reasoning, 9, 1-35 (1993)
4. Shafer, G.: A mathematical theory of evidence. Princeton University Press, Princeton, 1976.
5. Bendjebbour, A., Delignon, Y., Fouque, L., Samson, V., Pieczynski, W.: Multisensor Images Segmentation Using Dempster-Shafer Fusion in Markov Fields Context. IEEE Trans. on Geoscience and Remote Sensing, 39(8), 1789-1798 (2001)
6. Foucher, S., Germain, M., Boucher, J.-M., Benié, G. B.: Multisource classification using ICM and Dempster-Shafer theory. IEEE Trans. on Instrumentation and Measurement, 51(2), 277-281 (2002)
7. Hégarat-Mascle, Le S., Bloch, I., Vidal-Madjar, D.: Introduction of neighborhood information in evidence theory and application to data fusion of radar and optical images with partial cloud cover. Pattern Recognition, 31(11), 1811-1823(1998)
8. Tupin, F., Maitre, H., Bloch, I.: A first step toward automatic interpretation of SAR images using evidential fusion of several structure detectors. IEEE Trans. on Geoscience and Remote Sensing, 37(3), 1327-1343 (1999)
9. Pieczynski, W., Benboudjema, D.: Multisensor triplet Markov fields and theory of evidence. Image and Vision Computing, 24(1), 61-69 (2006)
10. Benboudjema, D., Pieczynski, W.: Unsupervised image segmentation using triplet Markov fields. Computer Vision and Image Understanding, 99(3), 476-498 (2005)
11. Boudaren, M. E. Y., An, L., Pieczynski, W.: Dempster-Shafer fusion of evidential pairwise Markov fields. International Journal of Approximate Reasoning, 74, 13-29 (2016)
12. Poggi, G., Scarpa, G., Zerubia, J. B.: Supervised segmentation of remote sensing images based on a tree-structured MRF model. IEEE Trans. on Geoscience and Remote Sensing, 43(8), 1901-1911 (2005)
13. Pieczynski, W., Tebbache, A.-N.: Pairwise Markov random fields and segmentation of textured images. Machine graphics and vision, 9, 705-718 (2000)
14. Li, S. Z.: Markov random field modeling in image analysis. Springer Science & Business Media, (2009)