UNSUPERVISED SEGMENTATION OF MULTISENSOR IMAGES USING GENERALIZED HIDDEN MARKOV CHAINS

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ABSTRACT

This work addresses the problem of unsupervised multisensor image segmentation. We propose the use of a recent method which estimates parameters of generalized multisensor Hidden Markov Chains. A Hidden Markov Chain is said to be "generalized" when the exact nature of the noise components is not known; we assume however, that each of them belongs to a finite known set of families of distributions. The observed process is a mixture of distributions and the problem of estimating such a "generalized" mixture contains a supplementary difficulty: one has to label, for each state and each sensor, the exact nature of the corresponding distribution. The general ICE-TEST method recently proposed allows one to solve such problems.

Key words : multisensor data, generalized mixture estimation, Hidden Markov Chains, Bayesian segmentation, unsupervised segmentation.

1. INTRODUCTION

Hidden Markov Chains are well known as an efficient tool for treating numerous concrete problems. Such models previously applied to speech processing problems and script recognition problems are here used in image segmentation. The image is transformed into a chain using the Hilbert-Peano curve [1]. This model was first proposed in [2] for the case of mono-sensor images and Gaussian mixtures. Nevertheless, the Gaussian model can be unsuited to describe reality and one has to consider the use of other noise types; moreover, it is desirable to be able to find automatically the correct nature of the noise for each class and each sensor. Such generalized mixtures have been previously studied assuming that the components lie in the Pearson system; in fact, an adaptation of the classical SEM algorithm ([3]) can be used to estimate such mixtures ([4]). In this paper we propose the use of a recent general method, called ICE-TEST, which allows one to find the noise components among any set of families of distributions ([5]). This method for estimating a generalized mixture is then coupled with a segmentation method, Maximum Posterior Mode (MPM), resulting in an unsupervised segmentation of multisensor images.
The organization of the paper is as follows:
The next section is devoted to the Hidden Markov Chain model in image segmentation. In the third section we briefly address the generalized mixture estimation problem and recall our particular method for Generalized Hidden Markov Chain model estimation, based on the Kolmogorov-Smirnov test. Section 4 is devoted to unsupervised segmentation and presents some results.

2. IMAGE SEGMENTATION USING HIDDEN MARKOV CHAINS

Let S be the set of pixels and $(s_1, \ldots, s_n)$ be pixels ordered according to the Hilbert curve given in Figure 1. $X = (X_{s\in S})$ is the class random field: thus each X_s takes its values in a finite set of classes $\Omega = \{\omega_1, \ldots, \omega_k\}$. $Y = (Y_{s\in S})$ is the observed field and each Y_s takes its values in $\mathbb{R}^m$ where m is the number of sensors; thus $Y_s = (Y_s^1, \ldots, Y_s^m)$. We assume that the observations for a given class in different sensors are independent. Considering $(X_1, \ldots, X_n) = (X_{s_1}, \ldots, X_{s_n})$ as a Hidden Markov Chain, the problem of image segmentation can be seen as the problem of the Hidden Markov Chain restoration.
A sequence of random variables $X = (X_n)_{n\in N}$ taking their values in $\Omega = \{\omega_1, \ldots, \omega_k\}$ is a Markov random chain if it verifies for every $n \geq 1$:

$$P(X_{n+1} = \omega_{i_{n+1}} \mid X_n = \omega_{i_n} \ldots X_1 = \omega_{i_1}) = P(X_{n+1} = \omega_{i_{n+1}} \mid X_n = \omega_{i_n}) \qquad (1)$$

Then, the distribution of $X = (X_n)_{n\in N}$ is given by the distribution of X_1, called the initial distribution, and a sequence of transition matrices $a_{ij} = P(X_{n+1} = \omega_j \mid X_n = \omega_i)$. In what follows we will assume that

$$c_{ij} = P(X_n = \omega_i, X_{n+1} = \omega_j) \quad (2)$$

does not depend on n. Thus the initial distribution is given by

$$\pi_i = P(X_1 = \omega_i) = \sum_{j=1}^{k} c_{ij} \quad (3)$$

and we have just one transition matrix $A = [a_{ij}]$, with

$$a_{ij} = \frac{c_{ij}}{\sum_{j=1}^{k} c_{ij}} \quad (4)$$

We will assume that the random variables $Y = (Y_s)_{s \in S}$ are independent conditionally on X and that the distribution of each Y_s conditional on X is equal to its distribution conditional on X_s. Thus all distributions of Y conditional on X are given by k distributions of Y_s conditional on $X_s = \omega_1, \ldots, \omega_k$, respectively. The latter distributions are given by densities $f_1, \ldots, f_k$ with respect to the Lebesgue measure and each f_i of densities, defined on $\mathbb{R}^m$, is given by m densities $f_i^1, \ldots, f_i^m$ on $\mathbb{R}$: $f_i(Y_s^1, \ldots, Y_i^m) = f_i^1(Y_i^1) \times \ldots \times f_i^m(Y_i^m)$. We denote by $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ the realizations of X and Y, respectively.
Let us consider the Forward-Backward probabilities which will be used in estimation and restoration stages:

$$F_t(\omega_i) = P[X_t = \omega_i, Y_1 = y_1, \ldots, Y_t = y_t]$$
$$B_t(\omega_i) = P[Y_{t+1} = y_{t+1}, \ldots, Y_n = y_n \mid X_t = \omega_i]$$

$$F_1(\omega_j) = N_1 \pi_j f_j(y_1)$$
$$F_t(\omega_j) = N_t \left(\sum_{i=1}^{k} F_{t-1}(\omega_i) a_{ij} \right) f_j(y_t) \quad \text{for} \quad t > 1 \quad (5)$$

$$B_n(\omega_i) = 1$$
$$B_t(\omega_i) = N_{t+1} \sum_{i=1}^{k} a_{ij} B_{t+1}(\omega_i) f_j(y_{t+1}) \quad \text{for} \quad t < n \quad (6)$$

Here N_t is a normalizing constant :

$$N_t = \left(\sum_{j=1}^{k} F_t(\omega_j) \right)^{-1}$$

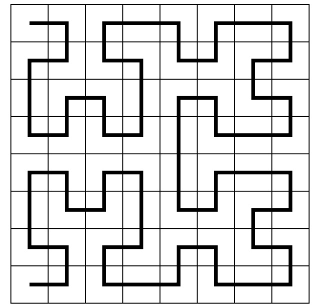


Figure 1: Hilbert scan

3. GENERALIZED MIXTURE ESTIMATION

The problem of mixture estimation is to find $\alpha = (\pi_i, a_{ij})$ and $f_1, \ldots, f_k$ from $Y = y$. In the "classical" mixture case the general form of $f_1, \ldots, f_k$ is known and these densities depend on a parameter β which has to be estimated from $Y = y$. For instance, if each f_i^l is Gaussian, β contains $k \times m$ means and $k \times m$ variances. In the "generalized" mixture case the general form of densities is not known exactly. However, the form of each f_i^l is in a given finite set of forms. Let $\Psi = \{F_1, \ldots, F_M\}$ be a set of families of distributions; we assume that each family F_j is characterized by the first four moments. Thus each f_i^l belongs to one of the families $F_1, \ldots, F_M$ and we do not know which one it is. The problem of finding the densities is then twofold: for each f_i^l, find the family F_j to which f_i^l belongs and find the parameter which fixes f_i^l in F_j. We use a general algorithm developed in [5], called ICE-TEST, based on the ICE algorithm ([6]), which in fact comprises a family of generalized mixture estimation methods. We propose the use of the Kolmogorov-Smirnov test and accordingly call this algorithm the ICE-KOLM algorithm. The iterative algorithm runs as follows:
At step q, let $\alpha^q = (\pi_i^q, a_{ij}^q)$ and $f_1^q, \ldots, f_k^q$ be current prior parameters and current densities. The updating procedure is:

- The initial distribution:

$$\pi_i^{q+1} = \frac{1}{n} \sum_{t=1}^{n} \sum_{j=1}^{k} \phi_t^q(i,j) \quad (7)$$

- the transition matrix:

$$a_{ij}^{q+1} = \frac{\sum_{t=1}^{n-1} \phi_t^q(i,j)}{\sum_{t=1}^{n-1} \sum_{j=1}^{k} \phi_t^q(i,j)} \quad (8)$$

where ϕ_t is defined with (5) and (6):

$$\phi_t(i,j) = \frac{P(X_t = \omega_i, X_{t+1} = \omega_j, Y = y)}{P(Y = y)} = \frac{F_t(\omega_i) a_{ij} f_j(y_{t+1}) B_{t+1}(\omega_j)}{\sum_{l=1}^{k} f_l(y_{t+1}) \sum_{j=1}^{k} F_t(\omega_j) a_{jl}} \quad (9)$$

- Simulate x^q, a realization of X, according to its α^q and $f_1^q, \ldots, f_k^q$ based distribution conditional on $Y = y$.
- for each sensor $l = 1, \ldots, m$
  - Calculate the conditional distribution parameters which depend on the first four moments and are estimated by empirical moments:

$$\hat{\mu}_i(x^q, y) = \frac{\sum_{t=1}^{n} y_t 1_{[x_t^q = \omega_i]}}{\sum_{t=1}^{n} 1_{[x_t^q = \omega_i]}} \quad (10)$$

$$\hat{\mu}_{i,p}(x^q, y) = \frac{\sum_{t=1}^{n} (y_t - \hat{\mu}_i)^p 1_{[x_t^q = \omega_i]}}{\sum_{t=1}^{n} 1_{[x_t^q = \omega_i]}} \quad 1 < p \leq 4 \quad (11)$$

  - perform M Kolmogorov tests which check the hypotheses

$$(H_0) \quad : \quad f_i^l = f \quad f \in \Psi = \{F_1, \ldots, F_M\}$$

$$(H_1) \quad : \quad f_i^l \neq f \quad f \in \Psi = \{F_1, \ldots, F_M\}$$

    Take $f_i^{l,q+1}$ as the density giving the positive answer to the test.
- For each class $i = 1, \ldots, k$ take $f_i^{q+1} = f_i^{1,q+1} \times \ldots \times f_i^{m,q+1})$ and update $(f_1, \ldots, f_k)$ with $(f_1^{q+1}, \ldots, f_k^{q+1})$.

4. UNSUPERVISED RESTORATION OF MARKOV CHAINS

4.1. Segmentation

At the end of the estimation stage, the prior distribution and conditional distributions have been calculated, so we can perform the segmentation. We have chosen the MPM algorithm which maximizes the marginal posterior probability:

$$\hat{X}_t = \omega_j \Leftrightarrow P(X_t = \omega_j \mid Y_1 = y_1 \ldots Y_n = y_n) = \max_{i \in \{1 \ldots k\}} P(X_t = \omega_i \mid Y_1 = y_1 \ldots Y_n = y_n) = \max_{i \in \{1 \ldots k\}} F_t(\omega_i) B_t(\omega_i) \quad (12)$$

The unsupervised restoration runs as follows:

- Estimate (π_i, a_{ij}, f_i) using the ICE-KOLM algorithm.
- For each $s = 1, \ldots, n$:
  - Calculate F_i, B_i using (5) and (6);
  - Classify $X_s = \omega_i$ where ω_i maximizes the posterior probability according to (12).

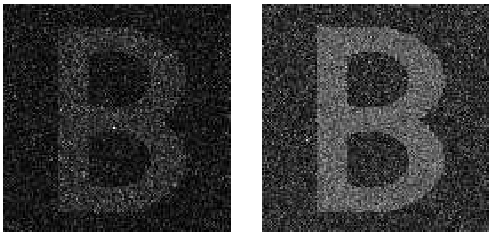
4.2. Numerical results

Let us consider the family $\Psi = \{F_1, F_2, F_3\}$ where F_1 are Gaussian distributions, F_2 are Gamma distributions and F_3 are Beta distributions of the first kind, plus a synthetic image "Letter B" with two sensors. Table 1 presents the distributions used to corrupt the image, the kind of distributions detected and the classification error rates after segmentation. Noisy images and segmented images are presented in Figures 2 and 3. We compare the ICE-KOLM algorithm with a classical one called ICE-GAUS [2] which assumes that all densities are Gaussian.

	Sensor 1	Sensor 2	τ_{MPM}
Real distributions	Gamma Gamma	Normal Gamma	0.89%
ICE-GAUS	Normal Normal	Normal Normal	3.7%
ICE-KOLM	Gamma Gamma	Normal Gamma	1.7%

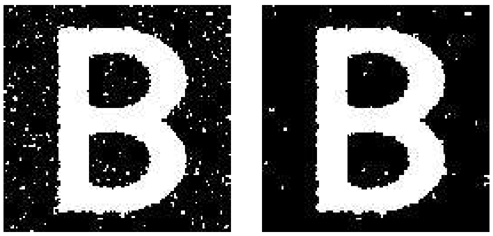
Table 1 : Density recognition with ICE-KOLM, real parameter based segmentation, ICE-GAUS based unsupervised segmentations, and ICE-KOLM based unsupervised segmentation.

In view of the results presented in Table 1 we observe that ICE-KOLM finds the correct distributions and the good estimation of the parameters improves the efficiency of the unsupervised Gaussian MPM restoration. Moreover, the error rates of the MPM based on the ICE-KOLM estimates are very close to the error rates of the MPM based on the true parameters.



sensor 1 sensor 2

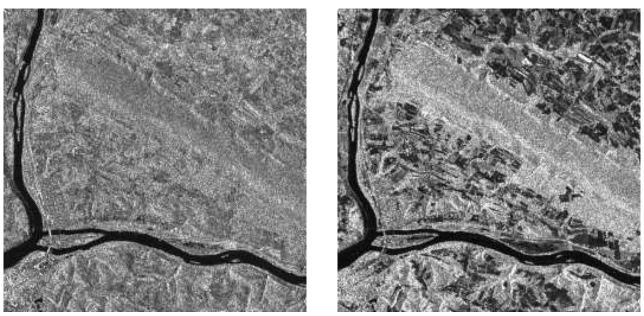
Figure 2: Noisy images in each sensor.



ICE-GAUS+MPM ICE-KOLM+MPM

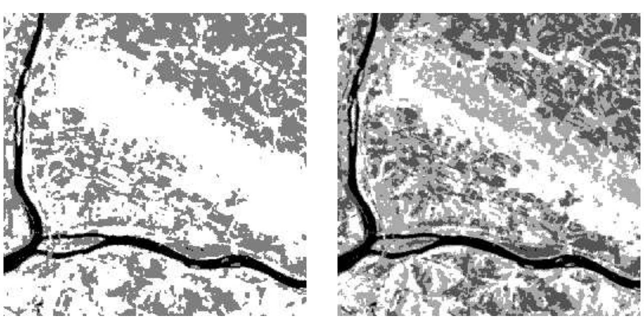
Figure 3: Segmented images.

We consider now a real multisensor radar image:



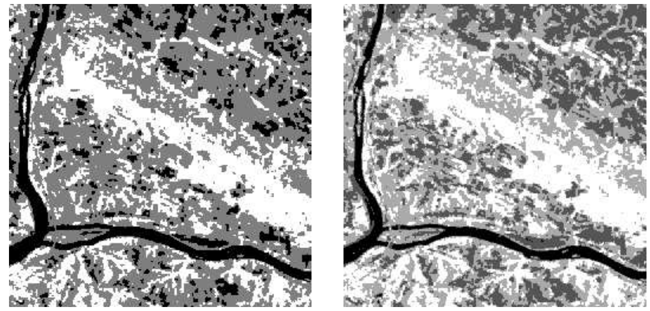
sensor 1 sensor 2

We present the result of two sensor, ICE-GAUSS and ICE-KOLM based-MPM segmentation:



3 classes 4 classes

Figure 4: ICE-GAUSS based MPM segmentation



3 classes 4 classes

Figure 5: ICE-KOLM based MPM segmentation

In case of three classes, the components detected by ICE-KOLM are Gamma, Gamma, Normal in sensor 1 and Gamma, Normal, Beta in sensor 2. In case of four classes, the components are Gamma, Normal, Normal, Normal in sensor 1 and Gamma, Beta, Beta, Normal in sensor 2. We may note that the nature of the components varies with the class. As the real nature of the ground is not avalaible, it is difficult to compare the efficiencies of the two methods. We observe, though, that fine details in image have been restored.

5. REFERENCES

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