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Statistical image segmentation using Triplet Markov fields

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ABSTRACT

Hidden Markov fields (HMF) are widely used in image processing. In such models, the hidden random field of interest $X=(X_s)_{s\in S}$ is a Markov field, and the distribution of the observed random field $Y=(Y_s)_{s\in S}$ (conditional on X) is given by $p(y|x)=\prod_{s\in S} p(y_s|x_s)$. The posterior distribution $p(x|y)$ is then a Markov distribution, which affords different Bayesian processing. However, when dealing with the segmentation of images containing numerous classes with different textures, the simple form of the distribution $p(y|x)$ above is insufficient and has to be replaced by a Markov field distribution. This poses problems, because taking $p(y|x)$ Markovian implies that the posterior distribution $p(x|y)$, whose Markovianity is needed to use Bayesian techniques, may no longer be a Markov distribution, and so different model approximations must be made to remedy this. This drawback disappears when considering directly the Markovianity of (X,Y); in these recent "Pairwise Markov Fields (PMF) models, both $p(y|x)$ and $p(x|y)$ are then Markovian, the first one allowing us to model textures, and the second one allowing us to use Bayesian restoration without model approximations.

In this paper we generalize the PMF to Triplet Markov Fields (TMF) by adding a third random field $U=(U_s)_{s\in S}$ and considering the Markovianity of (X,U,Y). We show that in TMF X is still estimable from Y by Bayesian methods. The parameter estimation with Iterative Conditional Estimation (ICE) is specified and we give some numerical results showing how the use of TMF can improve the classical HMF based segmentation.

Keywords: Hidden Markov field, Pairwise Markov field, Triplet Markov field, Iterative Conditional Estimation, statistical image segmentation, unsupervised classification.

1. INTRODUCTION

The modeling by hidden Markov fields (HMF) is widely used in various image processing problems. It consists of considering of two stochastic fields $X=(X_s)_{s\in S}$ and $Y=(Y_s)_{s\in S}$, where the unobservable realizations $X=x$ are of interest and have to be estimated from the observed $Y=y$. In this paper we will focuse on the image segmentation problem and so we will consider that each X_s takes its values in a finite set of classes $\Omega=\{\omega_1,...,\omega_k\}$, and each Y_s takes its values in the set of real numbers R. The spelling "hidden Markov" means that the hidden process X has a Markov distribution. When the distributions $p(y|x)$ of Y conditional on $X=x$ are simple enough, the pair (X,Y) keeps the same Markovian form of distribution, and it is the same for the distribution $p(x|y)$ of X conditional on $Y=y$. The Markovianity of $p(x|y)$ is crucial because it allows one to estimate the unobservable $X=x$ from the observed $Y=y$ even in the case of very large set S. One possible form of $p(y|x)$, which is frequently used in practice, is $p(y|x)=\prod_{s\in S} p(y_s|x_s)$ [1, 3, 11, 16, 19, 20, 21, 31]. In spite of its simplicity segmentation results obtained are correct in numerous situations, which indicate a good robustness of such HMF. However, when dealing with images containing numerous textures, such a simple form of $p(y|x)$ turns out to be in sufficient. In fact, each of textures has to

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be modeled by a model taking spatial correlation of the variables $(Y_s)_{s\in S}$ into account. One possible way it to model each texture by a Gaussian Markov field. So, for k classes we have k Gaussian Markov fields $Y^{\omega_1} = (Y_s^{\omega_1})_{s\in S}$, …, $Y^{\omega_k} = (Y_s^{\omega_k})_{s\in S}$, where the distribution of $Y^{\omega_1} = (Y_s^{\omega_1})_{s\in S}$ is the distribution of Y conditional on $X = x^{\omega_1} = (x_s = \omega_1)_{s\in S}$, …, and the distribution of $Y^{\omega_k} = (Y_s^{\omega_k})_{s\in S}$ is the distribution of Y conditional on $X = x^{\omega_k} = (x_s = \omega_k)_{s\in S}$. The observation field is then written $Y = (Y_s)_{s\in S}$, with $Y_s = Y_s^{X_s}$. Such a model is intuitively satisfactory and can be simulated; however, although $p(x)$ and $p(y|x)$ are Markov distributions, neither $p(x,y) = p(x)p(y|x)$ nor $p(x|y) = p(x)p(y|x)/p(y)$ is a Markov distribution. Roughly speaking, the introduction of the Markovianity of $p(y|x)$ removes the Markovianity of $p(x|y)$. This phenomenon has been pointed out in numerous papers [9, 12, 23], with a recent synthesis presented in [18]. It is a drawback because the use of Bayesian methods is not possible in the strict frame of the model and the latter has to be simplified. To remedy this, an original model called Pairwise Markov Field (PMF) has been proposed in [23]. In a PMF model one directly assumes the Markovianity of (X,Y). The latter implies the Markovianity of $p(y|x)$, which allows one to model textures, and the Markovianity of $p(x|y)$, which allows one to use different Bayesian segmentation methods. However, a PMF is not necessarily a HMF because $p(x)$ is not necessarily a Markov distribution.

In this paper, we propose to generalize PMF to more general models called "Triplet Markov Fields" (TMF). The generalization consists of introducing a third process $U = (U_s)_{s\in S}$ and considering that (X,U,Y) is a Markov field. The process U has not necessarily a physical existence and the problem remains the same as above: estimate $X = x$ from $Y = y$. The Triplet models are more general that the Pairwise models because the distribution of (X,Y), which is a marginal distribution of (X,U,Y), is not necessarily a Markov distribution. However, we show that the classical Bayesian segmentation methods remain workable in the TMF context. Some first experiments, performed in a supervised and an unsupervised way, show some situations in which the new model is of interest.

The paper is organized as follows. The next section is devoted to recalling the classical Hidden Markov Field (HMF) model. Triplet Markov Field (TMF) model is presented in Section 3 and the possibilities of Bayesian segmentation of images image using TMF are discussed, with some results related to simulated images. Section 4 is devoted to the parameter estimation problem. We specify how the general parameter estimation method called Iterative Conditional Estimation can be used in TMF context and some unsupervised segmentation results, concerning a simulated image images and a real radar one, are described. The last Section 5 contains some concluding remarks and perspectives for further works.

2. HIDDEN MARKOV FIELDS

Let S be the set of pixels, and X, Y two stochastic processes defined on S introduced above. The variables X_s and Y_s take their values in $\Omega = \{\omega_1, ..., \omega_k\}$, and R, respectively. The problem of the statistical segmentation is the problem to recover unobservable $X = x$ from the observed $Y = y$. Let us assume that the distribution of X is a Gibbs distribution with respect to the neighboring system $(A_s)_{s\in S}$ (for example, A_s is the set of four nearest neighbors of $s \in S$). Denoting by C the set of cliques (a clique being either a singleton or a set of pixels mutually neighbors), the ditribution of X is then written :

$$p(x) = \gamma \exp\left[-\sum_{c\in C} \varphi_c(x_c) \right] \tag{2.1}$$

X is then a Markov field with respect to $(A_s)_{s\in S}$; it is to say, it verifies $p(x_s | x_q, q \neq s) = p(x_s | x_q, q \in A_s)$. Furthermore, let $p(y|x)$ be of the form $p(y|x) = \prod_{s\in S} p(y_s|x_s)$. The distribution $p(x|y)$ is then classically written

$$p(x|y) = \gamma(y)\exp\left[-\left(\sum_{c\in C}\varphi(x_c) - \sum_{s\in S} Log(p(y_s|x_s))\right)\right] \quad (2.2)$$

Important is that the distribution of X conditional on $Y = y$ is still a Gibbs distribution. The latter allows us sampling of realizations of X according to this distribution (one can use Gibbs sampler or Metropolis algorithm), which makes possible the estimation of the marginal distributions $p(x_s = \omega_i|y)$. The latter having been estimated, one can perform the Bayesian MPM segmentation given by

$$\hat{s}_{MPM}(y) = (\hat{x}_s)_{s\in S} \text{, with } \hat{x}_s = \arg\max_{\omega\in\Omega} p(x_s = \omega|y) \quad (2.3)$$

3. TRIPLET MARKOV FIELDS

3.1 General Properties

Let S be the set of pixels, and X, U, Y three stochastic processes defined on S. For each $s \in S$, the variables X_s, U_s, and Y_s take their values in $\Omega = \{\omega_1,...,\omega_k\}$, $\Lambda = \{\lambda_1,...,\lambda_m\}$, and R, respectively. Furthermore, let $T = (X,U,Y)$, $Z = (X,Y)$, and $V = (X,U)$ be the stochastic processes linked with X, U, and Y. As above, the problem is to recover unobservable $X = x$ from the observed $Y = y$. Let us assume that the distribution of $T = (X,U,Y)$ is a Gibbs distribution with respect to the neighboring system. Denoting by C the set of cliques the ditribution of T is then written :

$$P_T(t) = \gamma\exp\left[-\sum_{c\in C}\varphi_c(t_c)\right] \quad (3.1)$$

as above, T is a Markov field with respect to $(A_s)_{s\in S}$: $p(t_s|t_q, q \neq s) = p(t_s|t_q, q \in A_s)$. One can then see that $p(v|y)$ is written

$$p(v|y) = \gamma(y)\exp\left[-\sum_{c\in C}\varphi(v_c, y_c)\right] \quad (3.2)$$

which still is a Markov distribution. As above, the latter implies that $p(v_s = (\omega_i,\lambda_j)|y)$ can be estimated from sampling of V, according to $p(v|y)$, in some way. The probabilities $p(v_s = (\omega_i,\lambda_j)|y)$ having been estimated, one calculates $p(x_s = \omega_j|y)$ by $p(x_s = \omega_j|y) = \sum_{\lambda\in\Lambda} p(x_s = \omega_j, u_s = \lambda|y)$. Finally, $p(x_s = \omega_j|y)$ are calculable and the formula (2.3) can be used to perform the Bayesian MPM segmentation method.

Example 3.1

Let U be a Markov field with respect to four nearest neighbors. We have then three kinds of cliques: singletons, pairs of pixels horizontally neighbors, and pairs of pixels vertically neighbors. The distribution $p(u)$ in then of the form (2.1). Let us assume that the functions φ_c in (2.1) are null on singletons and equal on other cliques, with $\varphi_c(u_s,u_r) = -\alpha$ if $u_s = u_r$, and $\varphi(u_s,u_r) = \alpha$ if $u_s \neq u_r$. Let $\Omega = \Lambda = \{\omega_1,\omega_2\}$ and let us assume that the random variables (Y_s) and the random variables (X_s) are independent conditionally on U, with $p(y_s|u) = p(y_s|u_i)$ and $p(x_s|u) = p(x_s|u_i)$. The distribution of T is written

$$p(t) = p(u)\prod_{s\in S} p(x_s|u_s)p(y_s|u_s) = \gamma \exp\left[-\left(\sum_{c\in C}\varphi(u_c) - \sum_{s\in S} Log(p(x_s|u_s)p(y_s|u_s))\right)\right] \quad (3.3)$$

We remark that $V=(X,U)$ is a classical hidden Markov field; however, as the distribution of X is a marginal distribution of $V=(X,U)$, it is not necessarily a Markov field, and thus (X,Y) it is not necessarily a hidden Markov field. Of course, if $p(x_s|u_s)=1$ for $x_s=u_s$, the processes X and U are equal and thus the TMF considered degenerates on a hidden Markov field.

Remark 3.1
We described above how HMF can be generalized to PMF, and how PMF can be generalized to TMF. An analogous manner allows one to generalize the Hidden Markov Chains (HMC) model to Pairwise Markov Chains (PMC) model [7, 8, 24], and to generalize the PMC to Triplet Markov Chain model [26, 27]. Furthermore, the so-called Hidden Markov Trees (HMT), which can appear as a fast concurrent to the HMF in image segmentation problems [15], can also be generalized to a Pairwise Markov Tree (PMT) model [25].

3. 2 Simulated image segmentation
Let us consider a TMF $T=(X,U,Y)$, with $\Omega=\{\omega_1,\omega_2\}$ and $\Lambda=\{\lambda_1,\lambda_2\}$. So, we have two real classes and two auxiliary ones. Let the distribution of $T=(X,U,Y)$ be defined by a Markov distribution of $V=(X,U)$, which will be assumed Markovian with respect to four nearest neighbors, and the distribution of Y conditional on V given by $p(y|v)=\prod_{s\in S} p(y_s|v_s)$. The random variables V_s can take four possible values $v_1=(\omega_1,\lambda_1)$, $v_2=(\omega_1,\lambda_2)$, $v_3=(\omega_2,\lambda_1)$, and $v_4=(\omega_2,\lambda_2)$, and thus we have four possible densities $p(y_s|v_s=v_1)$, $p(y_s|v_s=v_2)$, $p(y_s|v_s=v_3)$, and $p(y_s|v_s=v_4)$. The latter densities, which will be assumed Gaussian and independent from $s\in S$, define thus the distributions $p(y|v)$.
Finally, the distribution of $T=(X,U,Y)$ is given by

$$p(t) = p(v)p(y|v) = \gamma \exp\left[-\sum_{c\in C}\varphi_c(v_c)\right]\prod_{s\in S} p(y_s|v_s) \quad (3.4)$$

We assume that functions in φ_c in (3.4) are null on singletons and equal on other cliques, with $\varphi_c(v_s,v_r)=-\alpha$ if $v_s=v_r$, and $\varphi_c(v_s,v_r)=\alpha$ if $v_s\neq v_r$.
We present two series of experiments. In the first one, we have two real classes $\Omega=\{\omega_1,\omega_2\}$, and two auxiliary classes $\Lambda=\{\lambda_1,\lambda_2\}$. The simulated images and their segmentation results are presented in Figure 1, with the error ratios in Table1. In the second one, we have three real classes $\Omega=\{\omega_1,\omega_2,\omega_3\}$, and two auxiliary classes $\Lambda=\{\lambda_1,\lambda_2\}$. The simulated images and their segmentation results are presented in Figure 2, with the error ratios in Table2.
We have performed numerous simulations and the results presented are chosen as representative of the study. As usual, the quality of segmentation depend of the signal to noise ratio. The case 1 in Fig. 1 is rather strongly noisy. According to Tab. 1, the TMF based MPM (TMFMPM) is significantly more efficient that the HMF based one (HMFMPM). In such cases the classical HMCMPM method is useless and TMCMPM must be used. Of course, such cases are maybe not so current in practice; however, the simulation results show that they can, theoretically, exist. Case 2 is less noisy and there are very weak noise in Case 3. Both HMFMPM and TMFMPM work better in these cases and, according to the Bayesian theory, TMFMPM always gives better results that HMFMPM. Let us notice that the two means of two Gaussian densities used in HMFMPM are the smallest and the largest one: they are 0 and 1 in the Case 1, 0 and 3 in the case 2, and 1 and 10 in the case 3. We have tried other means, which can improve the efficiency of HMFMPM; however, TMFMPM remains more efficient. Same kind of remarks can be made concerning the three real classes, Fig. 2. The case 2 is very strongly noisy and both HMFMPM and TMFMPM give poor results. The medium case 2 gives medium results and a medium difference between HMFMPM and TMFMPM.

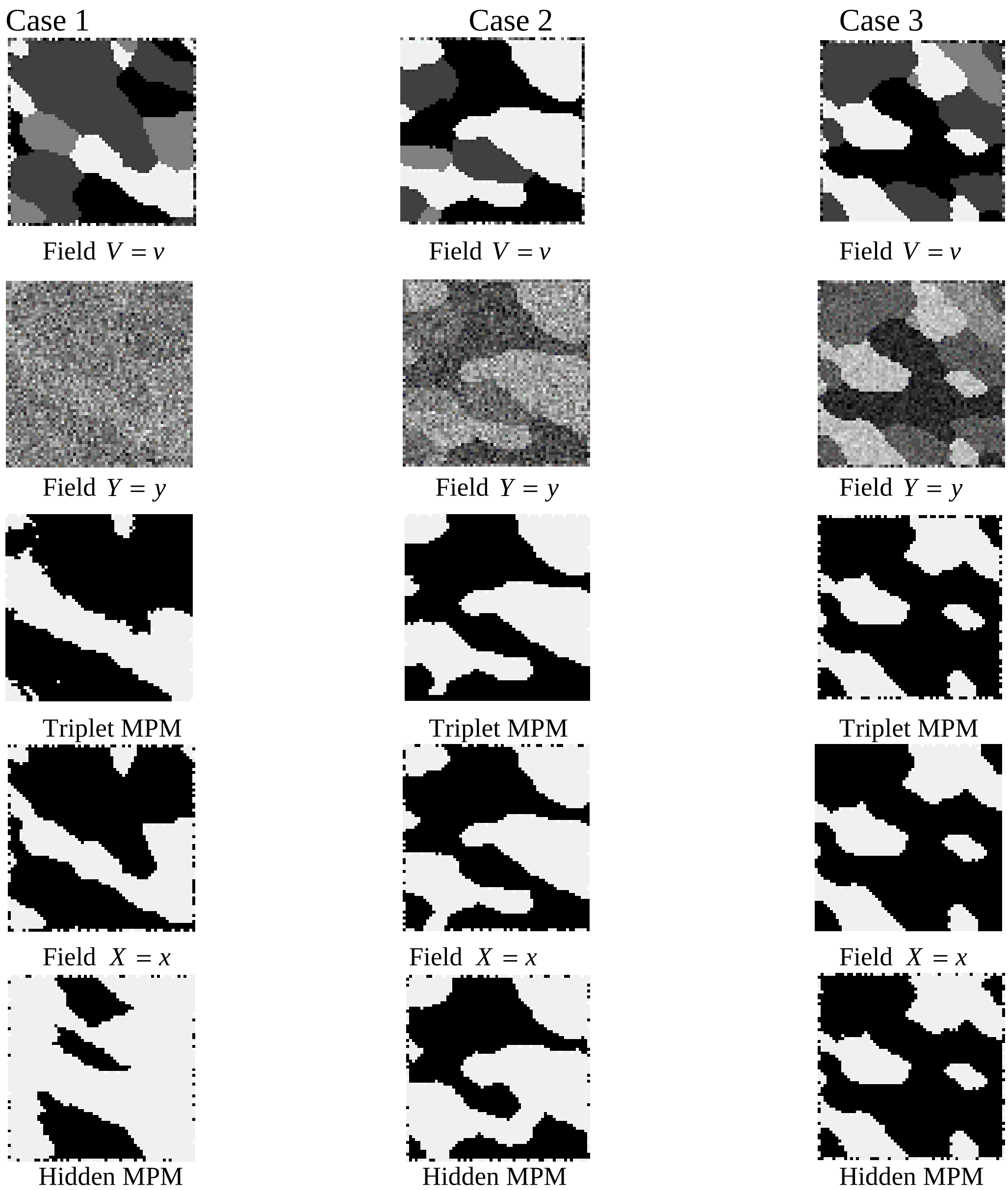


Fig. 1 : Three realizations of $V = v$ according to $p(v) = \gamma \exp[-\sum_{c \in C} \varphi_c (v_c)]$, with φ_c defined by $\alpha = 1$. All variances equal to 1, four means specified in Tab. 1.

Four means	Case 1 0,0.25,0.75,1	Case 2 0,1,2,3	Case 3 1,4,7,10
Error ratio Triplet MPM	14.57%	4.95%	2.97%
Error ratio Hidden MPM	44.55%	8.42%	3.49%

Tab. 1 : Error ratios of the three noise levels considered.

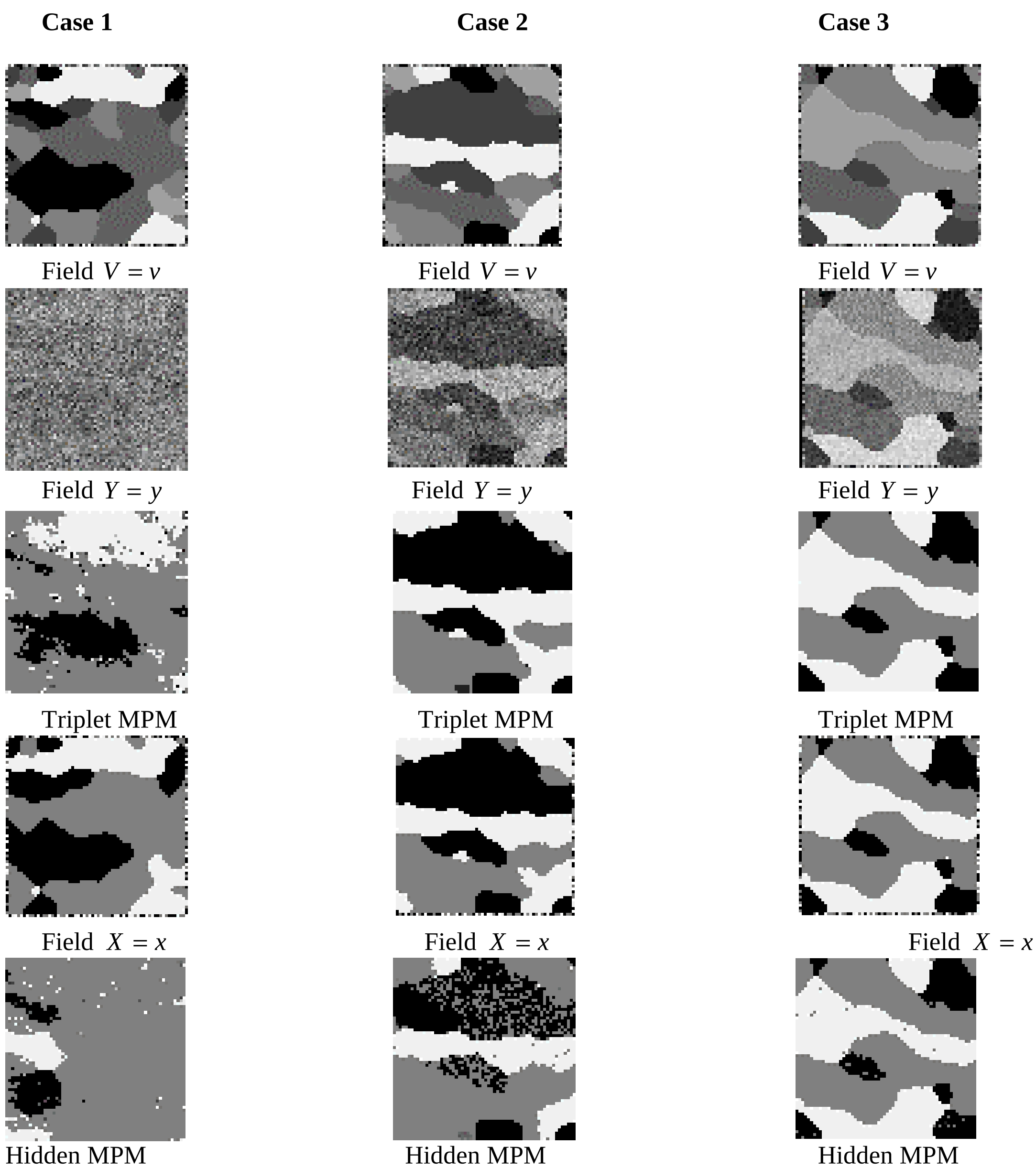


Fig. 2 : Three realizations of $V = v$ according to $p(v) = \gamma \exp[-\sum_{c \in C} \varphi_c(v_c)]$, with φ_c defined by $\alpha = 1$. All variances equal to 1, six means specified in Tab. 1.

Six means	Case 1 0,0.2,0.4,0.6,0.8,1	Case 2 0,1,2,3,4,5	Case 3 0,3,6,9,12,18
Error ratio Triplet MPM	35.71%	8.49%	4.10%
Error ratio Hidden MPM	50.26%	27.27%	5.05%

Tab. 2 : Error ratios of the three noise levels considered.

4. PARAMETER ESTIMATION

The problem of parameter estimation from the only observed data $Y = y$ becomes a crucial one when wishing to propose unsupervised processing. It is a well known and difficult general problem and one possible solution widely used is the "Expectation-Maximization" (EM) method [17]. For example, EM is very efficient in classical hidden Markov chains model, especially when the noise is Gaussian. Its use in the hidden Markov fields context remains possible; however, its implementation is generally more difficult [4, 17, 32] - mainly because the likelihood is difficult to handle – and some alternative methods have then been proposed [9, 21, 22, 31]. We choose here to describe the so-called "Iterative Conditional estimation" (ICE), which is fairly general and flexible method. Firstly proposed in [22], ICE has been successfully used in different applications of hidden Markov models to different image processing problems [10, 13, 14, 19, 20]. Furthermore, first applications of ICE to Pairwise Markov Chains (PMC) and Pairwise Markov Fields have also given promising results [7, 8, 29]. ICE resembles EM and some relationships are specified in [6]. So, we briefly discuss how the particular ICE method used in [2, 19] can be adapted to the Triplet models

4.1 Iterative Conditional Estimation

Let us consider stochastic processes $V = (V_1,...,V_n)$, $Y = (Y_1,...,Y_n)$, and $T = (V,Y)$. Let P_T^θ, the distribution of T, depend on a parameter $\theta \in \Theta$. The problem is to estimate θ from a sample $y = (y_1,..., y_n)$. Iterative Conditional Estimation (ICE) is based on the following assumptions :

(i) there exists an estimator of θ from the complete data: $\hat{\theta} = \hat{\theta}(t) = \hat{\theta}((v_1, y_1),...,(v_n, y_n))$;

(ii) for each $\theta \in \Theta$, either the conditional expectation $E_\theta[\hat{\theta}(T)|Y = y]$ is computable, or simulations of V according to its distribution conditional to $Y = y$ are feasible.

ICE is an iterative method which runs as follows :

1. Initialize $\theta = \theta^0$;
2. for $q \in N$,

-put $\theta^{q+1} = E_{\theta^q}[\hat{\theta}(T)|Y = y]$ if the conditional expectation is computable;

- if not, simulate l realizations $v^1,...,v^l$ of V (each v^i is a sequence) according to its distribution conditional to $Y = y$ and based on θ^q and put $\theta^{q+1} = \frac{\hat{\theta}(v^1, y) + ... + \hat{\theta}(v^l, y)}{l}$.

Remark 4.1

Let us recall that if P_T^θ admits a density f_T^θ with respect to some measure, and if the Maximum Likelihood estimator $\hat{\theta}_{ML}(t) = \arg\max_\theta \log(f_T^\theta(t))$ exists, the well known EM procedure would be:

1. Initialize $\theta = \theta^0$;
2. for $q \in N$, put $\theta^{q+1} = \arg\max_\theta E_{\theta^q}[\log(f_T^\theta(t)|Y = y]$

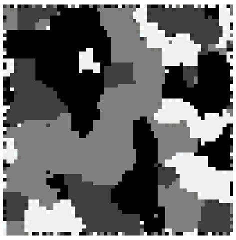
We can see that ICE is more general because any estimator $\hat{\theta} = \hat{\theta}(t)$, which can possibly be $\hat{\theta}_{ML}(t)$, can be used, and it also often is more flexible, because it is often easier, at least in the Markov field case considered here, to simulate realizations of V than to search the maximum of a complex function. We also see that when we take in ICE $\hat{\theta} = \hat{\theta}_{ML}$ and when the operations "argmax" and "expectation" can be inverted, ICE and EM are the same procedure.

4.2 ICE in Gaussian TMF

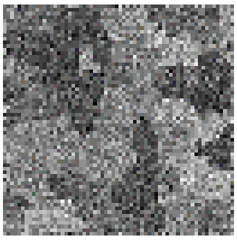
Let $Z = (X,Y)$ be a TMF, with $T = (V,Y)$ a PMF. Knowing that in a TMF the distribution of V conditional on $Y = y$ is a Markov field distribution, its simulations are feasible using the Gibbs sampler or Metropolis algorithm, and thus the condition (ii) is always verified. So, ICE is workable in a TMC once we have an estimator $\hat{\theta} = \hat{\theta}(t)$. In the TMF we use, $T = (V,Y)$ is a classical hidden Markov field and thus different classical estimators, like Coding estimator [1], Stochastic Gradient, or still Least Square estimator [9]. We chose here to the Last Square (LS) estimator because it is rather a fast one and it gave good results, when associated to ICE, in [2, 19]. Let us slightly complicate the distribution $p(v) = \gamma \exp[-\sum_{c \in C} \varphi_c(v_c)]$ of the Markov field by introducing two different parameters : for (s,t) horizontal neighbors we have $\varphi_c(v_s, v_r) = -\alpha_1$ if $v_s = v_r$, and $\varphi_c(v_s, v_r) = \alpha_1$ if $v_s \neq v_r$, and the same for perpendicular neighbors, with α_2 instead of α_1. So, for two real classes $\Omega = \{\omega_1, \omega_2\}$, and two auxiliary classes $\Lambda = \{\lambda_1, \lambda_2\}$, we have to estimate the parameters $\alpha = (\alpha_1, \alpha_2)$, four means, and four variances of the four Gaussian distributions in (2.3).

Finally, if we designate by β the four means and four variances to be estimated, the estimator $\hat{\theta} = \hat{\theta}(t)$ $\hat{\theta} = \hat{\theta}(t) = \hat{\theta}(v, y) = (\hat{\alpha}(v), \hat{\beta}(v, y))$, where $\hat{\alpha}(v)$ is the LS estimator and $\hat{\beta}(v, y)$ simply is composed by the empirical means and variances (the four subsets S_1, …, S_4 of the set of pixels S defined by $S_1 = \{s : v_s = v_1\}$, …, $S_4 = \{s : v_s = v_4\}$ being known, the mean m_i and variance σ_i^2 corresponding to v_i are $\hat{m}_i = \frac{1}{Card(S_i)} \sum_{s \in S_i} y_i$ and $\hat{\sigma}_i^2 = \frac{1}{Card(S_i)} \sum_{s \in S_i} (y_i - \hat{m}_i)^2$). The initialization of ICE is made using the cumulative histogram H . Four subsets G_1, …, G_4 of the set G of gray levels are defined by $G_1 = \{g \in G / 0 \leq H(g) \leq 0.25\}$, $G_2 = \{g \in G / 0.25 \leq H(g) \leq 0.5\}$, $G_3 = \{g \in G / 0.5 \leq H(g) \leq 0.75\}$, and $G_4 = \{g \in G / 0.75 \leq H(g) \leq 1\}$ and used to obtain a four class v_1, …, v_4 image v^0 by $[v_s^0 = v_i] \Leftrightarrow [y_s \in G_i]$. The image v^0 is then used to obtain $\alpha^0 = \hat{\alpha}(v^0)$ and $\beta^0 = \hat{\beta}(v^0, y)$, which gives the initialization $\theta = \theta^0$.

We performed numerous simulations and TMF-ICE-MPM almost always works better than HMF-ICE-MPM. One among the most striking results is presented in Fig. 3.



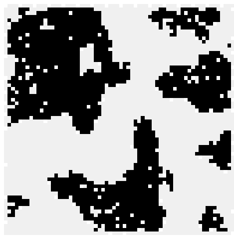
Four class image



Noisy image

Real two class image

TMF-ICE-MPM
Error ratio : 6,42 %



HMF-ICE-MPM
Error ratio : 11,77 %

Fig. 3 : Simulated TMF, unsupervised segmentation, and error ratios. Real and estimated parameters in Tab. 3

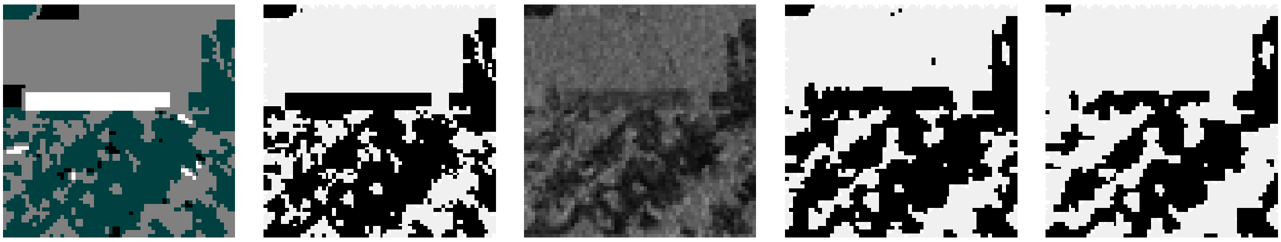
Real means	0,00	1,00	2,00	3,00
Estimated means	0, 48	0,94	01,97	2,99
Real variances	1,00	1,00	1,00	1,00
Estimated variances	1, 04	1, 03	1, 01	1,03
Real $\alpha = (\alpha_1, \alpha_2)$	(1, 1)			
Estimated $\alpha = (\alpha_1, \alpha_2)$	(0,80;0,92)			

Tab. 3 Real and estimated parameters of TMF in Fig. 3

3.3 Real radar image segmentation

Let us consider a real radar image presented in Fig., Im. 3. This image has been studied in [5]. We know that there are four classes "Cultivation"= $v_1 = (\omega_1, \lambda_1)$, "Recent pasture"= $v_2 = (\omega_1, \lambda_2)$, "Dense forest"= $v_3 = (\omega_2, \lambda_1)$, and "Burnt plot"= $v_4 = (\omega_2, \lambda_2)$. Let us assume that we are interested on two class segmentation : "Human cultures", and "Other". So, "Human cultures"=("Cultivation" or "Recent pasture")= ω_1, , and "Other" =("Burnt plot" or "Dense forest")= ω_2.

The results of TMF and HMF based segmentation into two classes are presented in Fig. 4, and the four Gaussian distributions estimated with ICE are given in Fig. 4.



Im. 1 Four class ground truth v_1, v_2, v_3, v_4.

Im. 2 Two class ground truth ω_1, ω_2.

Im. 3 Observed image

Im. 4 TMF-ICE-MPM Error ratio : 11,97 %

Im. 5 HMF-ICE-MPM Error ratio : 15,80 %

Fig. 4 : Real radar image, its ground truth, and TMF-ICE-MPM, HMF-ICE-MPM TMF, segmentation resuts.

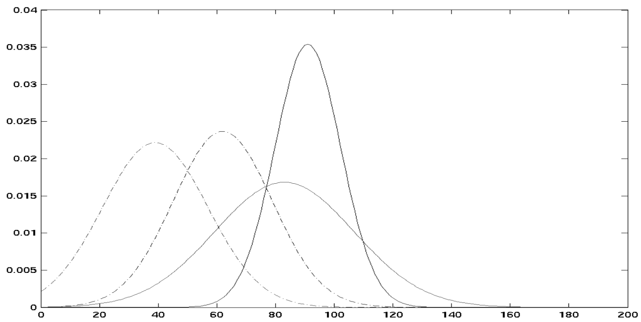


Fig. 5 : Four Gaussian distributions (“Cultivation”, “Recent pasture, “Dense forest”, and “Burnt plot”, respectively) estimated with ICE from Im. 3, Fig. 4. Estimated $\alpha = (\alpha_1, \alpha_2)$ is (0,86; 0,85).

5. CONCLUSION

We presented in this paper a new model called “Triplet Markov Fields” (TMF). The basic idea is the same that in the recent “Triplet Markov Chains” (TMC) model [26, 27]. For the observed random field Y and the searched random field X, it consists on introducing an auxiliary random field U and on considering the Triplet $T = (X, U, Y)$ as a Markov field. Such models are very rich and flexible, because there are little constraints in choosing U. Furthermore, the standard estimation procedures like Iterative Conditional Estimation (ICE) and the standard Bayesian segmentation methods like Maximum Posterior Mode (MPM) are workable in the TMF context. Some experiments have been described and the general conclusion is that TMF can be of interest with respect to the well known HMF. Furthermore, the latter appears as a very particular case of the former.

A real radar image, in which each of two searched classes contains two subclasses, has also been considered and the Bayesian MPM method based on the new model turns out to work better than the same method based on the classical HMF model.

We can view two possible directions for further developments : (i) more complex situations, in which the noise is not known and can possibly be non Gaussian (which frequently occurs in radar images [5], among others), could be considered (such situations have been studied in the context of HMF in [4, 28], and in the context of PMC in [7, 8], (ii) extensions of general hidden Graphical models [29] to Triplet Graphical models.

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