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UNSUPERVISED SEGMENTATION OF NON STATIONARY IMAGES WITH NON GAUSSIAN CORRELATED NOISE USING TRIPLET MARKOV FIELDS AND THE PEARSON SYSTEM

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ABSTRACT

The hidden Markov field (HMF) model has been used in many model-based solutions for image segmentation, and generally gives satisfying results. However, when the class image is non stationary, the unsupervised segmentation results provided by HMF can be poor. In this paper, we propose a new model based on triplet Markov fields (TMF) and Pearson system which enables one to deal with non stationary hidden fields and correlated, possibly non Gaussian noise. Moreover, the nature of marginal distributions of the noise can vary with the class. We specify a new general parameter estimation method and apply it to unsupervised Bayesian image segmentation.

1. INTRODUCTION

Hidden Markov fields (HMF) and Bayesian segmentation based on them, can be of outstanding efficiency when dealing with the important problem of unsupervised image segmentation. In such models, we have the hidden Markov field $X=(X_s)_{s\in S}$ and the observed one $Y=(Y_s)_{s\in S}$, and the problem is to estimate $X=x$ from $Y=y$. The simplest models, in which X is a Markov field and the random variables (Y_s) are independent conditionally on X, can give good results in many situations; however, they turn out to be too simple when considering complex images (non stationary, textured, strongly noisy, …). A pairwise Markov field (PMF) model has then been proposed, which consists in directly considering that the pair $Z=(X,Y)$ is a Markov field [11]. This implies that both conditional distributions $p(y|x)$ and $p(x|y)$ are Markov : the former fact allows one to better model complex noises, and the latter one still allows one to apply Bayesian segmentation. Afterwards, triplet Markov fields (TMF) have been proposed, in which one introduces a third random field $U=(U_s)_{s\in S}$ and assumes the Markovianity of the triplet $T=(X,U,Y)$ [10]. TMF can then be applied in numerous situations, with different interpretations for the third field U. In particular, one possible meaning for $U=(U_s)_{s\in S}$ is to assume that $U=u$ defines different homogeneities of (X,Y). This means that the Markov field distribution $p(x,y|u)$ is a non-stationary one [2] (let us also notice a recent Markov tree based model allowing one to deal with non stationary images [9]).

Otherwise, an important problem is to manage non Gaussian and correlated noise. In fact, such noises occur in many situations, like those related to sonar images or to radar ones ([6], [7], among others). This has not been solved, to our knowledge, in the hidden Markov fields context and we thus propose here a new model which extends the model proposed in [2]. Moreover, we propose a "generalized mixture" estimation method, which means that the very nature of the conditional marginal distributions $p(y_s|x_s)$ are not know exactly and can vary with the class x_s. Such methods have already been proposed in the case of classical HMF [4], hidden Markov chains[5], and hidden Markov trees[8]. Therefore, here we extend the methods in [4] to the TMF considered.

Finally, the paper contains the following contributions :

(i) the triplet Markov fields used in non stationary images presented in [2] are extended to the general case where the noise can be correlated and its marginal distributions $p(y_s|x_s)$ can be of any form and can vary with the class x_s;

(ii) a new model identification method, which is a "generalized mixture" estimation method based on the Pearson system is proposed, and validated via experiments.

2. MODELLING NON STATIONARY IMAGES WITH TRIPLET MARKOV FIELDS

2.1. Gaussian noise

Let us shortly recall the model proposed in [2]. First, let us consider the very classical Markov field $X = (X_s)_{s\in S}$ with whose distribution is defined by the energy

$$W(x) = \sum_{(s,t)\in C_H} \alpha_H \left(1 - 2\delta(x_s, x_t)\right) + \sum_{(s,t)\in C_V} \alpha_V \left(1 - 2\delta(x_s, x_t)\right) \quad (2.1)$$

where C_H is the set of couples of pixels horizontally neighbors, C_V is the set of couples of pixels vertically neighbors, and $\delta(x_s, x_t)$ verifies $\delta(x_s, x_t) = 1$ for $x_s = x_t$, and $\delta(x_s, x_t) = 0$ for $x_s \neq x_t$. Furthermore, let us consider classically that the random variables $(Y_s)_{s\in S}$ are independent conditionally on $X = (X_s)_{s\in S}$. The distribution of the classical HMF (X, Y) is then written

$$p(x, y) = \gamma \exp[-W(x) + \sum_{s\in S} Log(p(y_s | x_s))] \quad (2.2)$$

Such HMF have been generalized in [2] by considering a Markov field (X, U), with two stationarities $\Lambda = \{a, b\}$, whose distribution is defined by the energy

$$\begin{aligned} W(x,u) = & \sum_{(s,t)\in C_H} \alpha_H^1 \left(1 - 2\delta(x_s, x_t)\right) - \\ & \left(\alpha_{aH}^2 \delta^*(u_s, u_t, a) + \alpha_{bH}^2 \delta^*(u_s, u_t, b)\right)\left(1 - \delta(x_s, x_t)\right) \quad (2.3) \\ & + \sum_{(s,t)\in C_V} \alpha_V^1 \left(1 - 2\delta(x_s, x_t)\right) - \\ & \left(\alpha_{aV}^2 \delta^*(u_s, u_t, a) + \alpha_{bV}^2 \delta^*(u_s, u_t, b)\right)\left(1 - \delta(x_s, x_t)\right), \end{aligned}$$

with $\delta^*(u_s, u_t, a) = 1$ for $u_s = u_t = a$, and $\delta^*(u_s, u_t, a) = 0$ otherwise and $\delta(u_s, u_t, b) = 1$ for $u_s = u_t = b$, and $\delta^*(u_s, u_t, b) = 0$ otherwise. We can easily verify that for $x = u$ the energy (2.3) is reduced to the energy (2.1); therefore the model (2.3) is a generalization of the classical model (2.1).
Finally, the distribution of (X, U, Y) is defined by

$$p(x, u, y) = \gamma \exp[-W(x, u) + \sum_{s\in S} Log(p(y_s | x_s))] \quad (2.4)$$

Both models HMF given by (2.2) and TMF given by (2.4) allows one to estimate $p(x|y)$. In HMF this is classically done from (2.2) using the Gibbs sampler, and in TMF this is done in two steps : (i) estimate $p(x_s, u_s | y)$ by the Gibbs sampler; (ii) calculate $p(x_s|y) = \sum_{u_s\in\Lambda} p(x_s, u_s | y)$. Therefore, the Bayesian Maximum Posterior Mode (MPM) can be used in both HMF and TMF given by (2.2) and (2.4), respectively.

2.2. Non Gaussian noise

In the Gaussian case above, let us consider a set of independent Gaussian variables $(Y_s')_{s\in S}$, with $E[Y_s'] = 0$, $Var[Y_s'] = 1$ for each $s \in S$, and such that $Y' = (Y_s')_{s\in S}$ is independent from $X = (X_s)_{s\in S}$ and $U = (U_s)_{s\in S}$. Then we can say that $Y = (Y_s)_{s\in S}$ is obtained from $Y' = (Y_s')_{s\in S}$ by $Y_s = \sigma_{X_s} Y_s' + m_{X_s}$.
The first idea of this paper is to use the same set of independent variables $Y' = (Y_s')_{s\in S}$ to obtain any other kind of distribution. In fact, putting F the cumulative function of the Gaussian distribution $N(0,1)$, and putting G the cumulative function of the desired distribution, we know that $Y_s = G^{-1} \circ F(Y_s')$ has the desired distribution G.
The second idea is to consider a Markov Gaussian field $Y' = (Y_s')_{s\in S}$ instead of the set of independent variables above.
Finally, the new model we propose is the following. Let (X, U) be a Markov field, and let $Y' = (Y_s')_{s\in S}$ be a Markov Gaussian field independent from (X, U) and such that $E[Y_s'] = 0$, and $Var[Y_s'] = 1$. For each class ω_i in $\Omega = \{\omega_1, ..., \omega_k\}$, let G_i be the cumulative function of the desired distribution $p(y_s | x_s = \omega_i)$, and let F be the cumulative function of the Gaussian distribution $N(0,1)$. Then the TMF we propose is $T = (X, U, Y)$, with $Y_s = G_{X_s}^{-1} \circ F(Y_s')$.
Therefore we obtain a model such that the random variables (Y_s) are correlated conditionally on X (the noise is correlated), and such that the marginal distributions $p(y_s | x_s = \omega_i)$ are not necessarily Gaussian (their shape can even vary with the class). For example, assuming that (X, U) is a Markov field defined by the energy (2.3), and assuming that the distribution of the Gaussian Markov field $Y' = (Y_{s'})_{s\in S}$ is given by the energy $\varphi(y') = \sum_{c\in C} \varphi_c(y_c')$, then the distribution of (X, U, Y) is given by

$$p(x, u, y) = \gamma \exp[-W(x, u) - \sum_{c\in C} \varphi_c(F^{-1} \circ G_{x_c}(y_c)) \quad (2.5)$$

where $F^{-1} \circ G_{x_c}(y_c) = (F^{-1} \circ G_{x_s}(y_s))_{s\in S}$.

3. MODEL IDENTIFICATION

In the first sub-section we assume that the shapes of $p(y_s|x_s = \omega_i)$ are known (they can vary with the class), and we describe how the parameter estimation method proposed in the Gaussian case in [1] can be extended to such a model. In the second sub-section we specify how the Pearson system can be used to search, in addition, a good shape of $p(y_s|x_s = \omega_i)$ for each class ω_i. This second case is an extension to the new model (non stationary hidden field and correlated noise) of the method presented in the case of classical HMF in [4].

3.1. Parameter estimation

Let us distinguish three kinds of parameters. We will designate by α the parameters defining the Markov distribution of (X, U), by β^1 the parameters defining the Markov Gaussian distribution of Y', and by β^2 the parameters defining the k cumulative functions G_1, …, G_k. Thus $\beta^2 = (\beta_1^2, ..., \beta_k^2)$, and each β_i^2 defines G_i. We will assume that we dispose of k estimators $\hat{\beta}_1^2$, ..., $\hat{\beta}_k^2$ such that each $\hat{\beta}_i^2$ estimates β_i^2 from samples produced by G_i.

Thus the problem is to estimate $\theta = (\alpha, \beta^1, \beta^2)$ from $Y = y$. The method we propose is an iterative one and extends to the proposed model the "Iterative Conditional Estimation" (ICE) based method described in [1].

Let θ^0 an initial value of *6*. The next value θ^{q+1} is defined from θ^q and $Y = y$ in the following way :

(i) simulate $x^q = (x_s^{q+1})_{s\in S}$ according to $p(x|y, \theta^q)$ by Gibbs sampler;

(ii) estimate α^{q+1} from x^q as explained in [2];

(iii) for each $i = 1, ..., k$, let $S_i^{q+1} = \{s \in S | x_s^q = \omega_i\}$. For each $i = 1, ..., k$, use the restriction of y to S_i^{q+1} and $\hat{\beta}_i^2$ to determinate $(\beta^2)^{q+1}$, which gives G_i^{q+1};

(iv) define $y' = (y_s')_{s\in S}$ by $y_s' = F^{-1} \circ G_{x_s^q}^{q+1}(y_s)$, and use y' to estimate $(\beta^1)^{q+1}$ by the method in [2] (recall that $y' = (y_s')_{s\in S}$ is a Gaussian Markov field).

3.2. Generalized mixture estimation with the Pearson system

Let us briefly recall what the Pearson system is and how the first four moments define a probability distribution in it.

The Pearson system is the set of probability densities f verifying

$$\frac{df(x)}{dx} = -\frac{x + a}{c_0 + c_1 x + c_2 x^2} f(x) \qquad (3.1)$$

where a, c_0, c_1, and c_2 are real parameters. When varying, these parameters define eight different shapes of f, called shape of "kind I", "kind II", and so on. Important is that these shapes can be found from the first four moments $\mu_1 = E[Y]$, $\mu_2 = E[(Y - E[Y])^2]$, $\mu_3 = E[(Y - E[Y])^3]$, $\mu_4 = E[(Y - E[Y])^4]$. Calculating the *"kurtosis"* $\gamma_1 = \frac{(\mu_3)^2}{(\mu_2)^3}$ and the *"skewness"* $\gamma_2 = \frac{\mu_4}{(\mu_2)^2}$, one considers

$$c = \frac{\gamma_1(\gamma_2 + 3)^2}{4(4\gamma_2 - 3\gamma_1)(2\gamma_2 - 3\gamma_1)(2\gamma_2 - 3\gamma_1 - 6)} \qquad (3.2)$$

and then the eight different shapes are given by :

type I : $c < 1$; type II : $\gamma_1 = 0$ and $\gamma_2 < 3$; type III : $2\gamma_2 - 3\gamma_1 - 6 = 0$; type IV : $0 < c < 1$; type V : $c = 1$; type VI : $c > 1$; type VII : $\gamma_1 = 0$ and $\gamma_2 > 3$; type VIII : $\gamma_1 = 0$ and $\gamma_2 = 3$. Otherwise, it can be shown that the type I corresponds to the beta distributions of the first kind, type III are gamma distributions, and type VIII are Gaussian distributions (the shapes of other types can be seen in [4]).

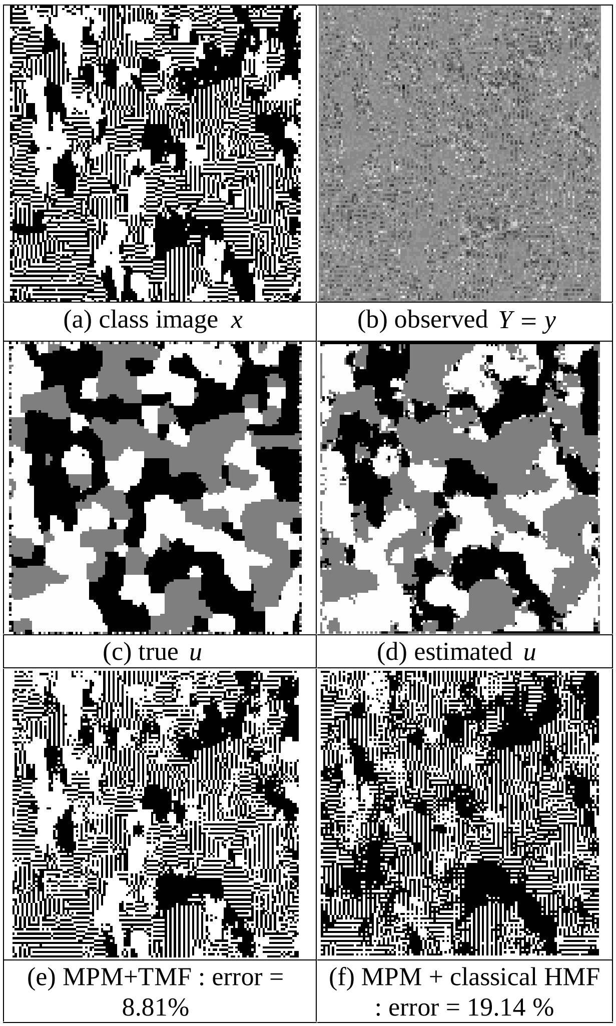
Therefore, knowing the first four moments we know what shape we are faced with and, in addition, these moments give the parameters of the corresponding distribution.

The Pearson system can then be used to estimate the following "generalized mixture" : let us assume that for each $i = 1, ..., k$ the distribution G_i is in the Pearson system, but we do not know what the shape of this distribution is. Such situations can occur in radar images. In fact, the shape of class distributions can vary with the class and, for a given class, it can even very with time [4]. Faced with such a situation, we can use in the method above the following estimators $\hat{\beta}_1^2$, ..., $\hat{\beta}_k^2$: each estimator $\hat{\beta}_i^2$ estimates the first four moments of G_i, which gives its shape and the parameters.

4. EXPERIMENTS

Let us consider the following example. Each X_s takes its values in the set of two classes $\Omega = \{\omega_1, \omega_2\}$, and each U_s takes its values in the set of three classes $\Lambda = \{a, b, c\}$ (there are three different homogeneities in the class image $X = x$). A realization (x, u) of the Markov field (X, U) is then simulated by Gibbs sampler and presented in (a), (c) in Figure 1 below. The class image is then noised according to

$p(y_s | x_s = \omega_1)$, which is a gamma distribution (type III in the Pearson system), and $p(y_s | x_s = \omega_2)$, which is a beta of the first kind distribution (type I in the Pearson system), More precisely, the parameters considered are $p(y_s | x_s = \omega_1) = \Gamma(1,2)$, and $p(y_s | x_s = \omega_2) = \mathrm{B}(2,1)$. Concerning the Gaussian Markov field Y', we have taken the following energy $\varphi(y') = \frac{1}{2}\left[\sum_{s \in S}(y_s')^2 + \sum_{(s,t)} -0.2 y_s' y_t'\right]$. The noisy image $Y = y$ is presented in (b), Figure 1, the estimated u, using the new model, is in (d). Finally, unsupervised segmentation results based on the new model and on the classical HMF are presented in (e) and (f).



(a) class image x | (b) observed $Y = y$ | (c) true u | (d) estimated u | (e) MPM+TMF : error = 8.81% | (f) MPM + classical HMF : error = 19.14 %

Figure 1. Class image x (a), its noisy version (b), u corresponding to x (c), estimated u (d), new method unsupervised segmentation result (e), and classical method one (f).

5. CONCLUSION

We have presented in this paper a new model and a new parameter estimation method. The model is a particular Triplet Markov field (TMF) allowing one to deal with hidden non stationary random field, with correlated and non Gaussian noise. We also proposed a parameter estimation method adapted to the new model and presented an example of an unsupervised image segmentation.

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