FUZZY STATISTICAL UNSUPERVISED IMAGE SEGMENTATION

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ABSTRACT

This paper deals with fuzzy Bayesian unsupervised image segmentation. At first, we introduce a new model and a method for its simulation.The images obtained that way are corrupted with Gaussian, white or correlated, noise. A blind Bayesian segmentation is performed using parameters estimated by the SEM algorithm adapted to our model. Finally this segmentation is compared with a classical method without taking into account the fuzzy class.

INTRODUCTION

In order to represent an image, to each pixel will be associated a numerical value or a label, which belongs to a finite set of classes. Those values are considered as a realization of a random field. The image is generally assumed to be homogeneous in each pixel, which is not always the case in reality. The "observed", or "noised" image is represented by another random field and the problem of segmentation is the problem of estimation of the "class" field from the "observed" field. Finally the image is modeled by two random fields: $\zeta=(\zeta_s)_{s\in S}$ (S is a set of pixels), representing the unobservable real image, and $X=(X_s)_{s\in S}$ the observed image. Since this inexactitude is an intrinsic propriety of the image, we propose a model including the possibility of "mixed" pixels using a fuzzy class in the case of binary images.

We consider the two "pure" classes ω_1, ω_2 . A classical model assumes that each ζ_s takes its value in $\{\omega_1, \omega_2\}$. Our approach supposes the existence of a third class, in addition to the two "pure" classes, the "fuzzy class" f_{12} assimilated to $]0,1[$. Thus each ζ_s takes its value in $\{\omega_1, \omega_2,]0,1[\}$. In the case of satellite data, if ω_1 represents "forest" and ω_2 " urban area" the fuzzy class designates pixels where houses and trees are simultaneously present. In this case the value of ζ_s is in $]0,1[$ and can bee seen as the proportion of the class ω_1.We propose a simulation of this model based on the Gibbs sampler.

The images obtained that way are corrupted by white and correlated Gaussian noises. The random field X takes its values in R and we assume to know the conditional density of X given ζ

We propose to use the blind Bayesian unsupervised segmentation method. The parameters required are estimated by the SEM algorithm adapted to our model. The results are compared with a classical equivalent method (without taking into account the fuzzy class).

THE FUZZY MODEL AND ITS SIMULATION

We express the law of each ζ_s as follow:

$$P_{\zeta_s} = \pi_1\delta_1 + \pi_2\delta_2 + (1- (\pi_1+\pi_2)) f . \mu$$

where $\pi_1=P[\zeta_s=\omega_1]$ $\pi_2=P[\zeta_s=\omega_2]$

δ_1 and δ_2 are the Dirac's weighed distributions

f is the density on $]0,1[$

μ is the Lebesgue measure

A The sampling of the random field ζ

This is performed in two steps:

1) Simulation of a three class random field using a Gibbs sampler ([3])
2) Simulation of the values on the fuzzy class

1) During the first step, the fuzzy class is considered as a real class: the random field ζ takes its values in $\{\omega_1,\omega_2,]0,1[\}$. The Markov assumption permits its simulation by a Gibbs sampler.

The potential function is defined only on the clique-pairs. The fuzzy class does not have the same behavior towards interactions as the two pure classes; therefore we need three differents parameters to define the potentiel function:

$$\Phi(s,t)=\begin{cases} -b \text{ if } (\zeta_s,\zeta_t)=(\omega_1,\omega_1) \text{ or} (\omega_2,\omega_2) \\ b \text{ if } (\zeta_s,\zeta_t)=(\omega_1,\omega_2) \text{ or } (\omega_2,\omega_1) \\ d \text{ if } (\zeta_s,\zeta_t)=(\omega_1,f_{12}) \text{ or } (\omega_2,f_{12}) \\ g \text{ if } (\zeta_s,\zeta_t)=(f_{12},f_{12}) \end{cases}$$

b defines the size of the two pure classes.
d manages the attraction between the fuzzy class and the other classes.
g acts on the proportion of the fuzzy class.

2) We define the density f conditionally to the four nearest neighbors and use an iterative procedure to obtain the realizations of the fuzzy class.
The visual appearence of the fuzzy class guides our choice of f. In some situations it models heterogeneous boundaries such a way that it appears between two pure zones and the proportion of the class ω_1 (respectively ω_2) increases near a pure zone of ω_1(respectively ω_2). Therefore we propose a density as follows:

$$f(x)=aSx+b$$

where • a is a parameter. When a increases f becomes a discrete distribution on $\{0,1\}$(which can be assimilated to $\{\omega_1,\omega_2\}$)

•$S\in[-1,1]$ and is an affine function of the sum of the values taken by ζ on the neighbourhood. S=-1 (resp. S=1) when the four neighbours of s take the value ω_1 (resp. ω_2)

• b is a normalizing constant

In each iteration the fuzzy pixels get values in]0,1[according to the law defined by the density f. Our goal is to simulate visually satisfactory images.

B Corrupted images

The images obtained above are corrupted by different Gaussian noises.
The distribution of X_s conditional to ζ_s is defined by:

$$P_{X_s}=\begin{cases} N(m_1,\sigma_1) \text{ if } \zeta_s=\omega_1 \\ N(m_1+(m_1-m_2)\varepsilon,\sigma_1+(\sigma_1-\sigma_2)\varepsilon) \text{ if } \zeta_s=\varepsilon\in]0,1[\\ N(m_2,\sigma_2) \text{ if } \zeta_s=\omega_2 \end{cases}$$

Correlated noises are simulated by taking the mobile averages computed in each pixel from the values in the neighborhood of a white noise. We propose two differents types of neighborhood: 8 or 24 neighbours. The pure classes are distinguished by (m_1,σ_1) and $(m_2\sigma_2)$ with possible combinations.

FUZZY UNSUPERVISED STATISTICAL SEGMENTATION

We adopt an unsupervised Bayesian blind segmentation. It consists of estimating the unobservable realization of ζ from the data X=x by ζ^* which maximizes the conditional distribution of ζ given X=x (the a posteriori distribution). This distribution can be expressed from the distribution P_{ζ_s} of ζ and the conditional distribution P_X^{ε} of X given ζ.

We suppose in this part that a=0, so f is taken uniform on]0,1[. Our classification runs in two steps:
a) We first apply a classical Bayesian classification to the three class random field (each ζ_s taking its values in $\{\omega_1, \omega_2,f_{12}\}$)
b) Pixels classified "fuzzy" are then re-classified according to the maximum of the posterior likelihood.

a) The first step comprises two differents parts:

(i) Estimation of the required parameters:
-of the distribution P_{ζ_s} of ζ_s : $\pi_1=P[\zeta_s=\omega_1]$ and $\pi_2=P[\zeta_s=\omega_2]$
-of the conditionnal distribution P_X^{ε} of X given ζ :
(m_1,σ_1^2) and (m_2,σ_2^2)

We use the SEM algorithm ([1],[4],[5]) in the case of the blind segmentation adapted to our model. It runs as follows:

•Giving initial values of the parameters

•In each iteration, for each observation x_i

*calculation of the conditional densities

$$f_1^k(x_i)=\frac{1}{\sqrt{2\pi}\sigma_1^k}\exp[-\frac{(x_i-m_1^k)^2}{2\sigma_1^{k\,2}}]$$

$$f_2^k(x_i)=\frac{1}{\sqrt{2\pi}\sigma_2^k}\exp[-\frac{(x_i-m_2^k)^2}{2\sigma_2^{k\,2}}]$$

$$F_{12}(x_i)=\int_0^1\frac{1}{\sqrt{2\pi}\sigma(x)}\exp[-\frac{(x_i-m(x))^2}{2\sigma(x)^2}]\,dx$$

where $m(x)=m_1+(m_1-m_2)x$ and $\sigma^2(x)=\sigma_1^2+(\sigma_2^2-\sigma_1^2)x$

*calculation of the a posteriori probabilities

$$p_1^k(x_i) = \frac{\pi_1^k f_1^k(x_i)}{\pi_1^k f_1^k(x_i) + \pi_2^k f_2^k(x_i) + \pi_{12}^k F_{12}^k(x_i)}$$

$$\text{where } \pi_{12}^k = 1-(\pi_1^k+\pi_2^k)$$

$$p_2^k(x_i) = \frac{\pi_2^k f_2^k(x_i)}{\pi_1^k f_1^k(x_i) + \pi_2^k f_2^k(x_i) + \pi_{12}^k F_{12}^k(x_i)}$$

Then we draw an element in the set $\{\omega_1, \omega_2, f_{12}\}$ according to the law defined by the above probabilities. We obtain a partition of the sample $x_1,\ldots,x_n$ $\{Q_1^k, Q_2^k, Q_{12}^k\}$

*Estimation of the parameters using the above partition

$$m_1^{k+1} = \frac{\Sigma x_{i,1}^k}{card(Q_1^k)} \qquad m_2^{k+1} = \frac{\Sigma x_{i,2}^k}{card(Q_2^k)}$$

$$\sigma_1^{k+1^2} = \frac{\Sigma(x_{i,1}^k - m_1^k)^2}{card(Q_1^k)} \qquad \sigma_2^{k+1^2} = \frac{\Sigma(x_{i,1}^k - m_2^k)^2}{card(Q_2^k)}$$

$$\pi_1^{k+1} = \frac{card(Q_1^k)}{n} \qquad \pi_2^{k+1} = \frac{card(Q_2^k)}{n}$$

When those values are steadying we obtain the estimators (m_1,σ_1^2), $(m_2\sigma_2^2)$, $\pi_1$and π_2.

(ii) Segmentation based on the above estimated parameters:

For each pixel s we estimate the realization of $\zeta_s$by the element of $\{\omega_1, \omega_2, f_{12}\}$ which maximises the a posteriori distribution computed from the estimated parameters.

$$\pi_1(x) = \frac{\pi_1 f_1(x)}{\pi_1 f_1(x) + \pi_2 f_2(x) + \pi_{12}\int_0^1 f_{12}(x,u)\, du}$$

$$\pi_2(x) = \frac{\pi_2 f_2(x)}{\pi_1 f_1(x) + \pi_2 f_2(x) + \pi_{12}\int_0^1 f_{12}(x,u)\, du}$$

b) In the second step the pixels classified "fuzzy" are re-classified by taking the value ε in $]0,1[$ which maximises the a posteriori density defined by:

$$f_{12}(\varepsilon,x_i) = 1_{]0,1[}(\varepsilon)\frac{1}{\sqrt{2\pi}\sigma(\varepsilon)}\exp[-\frac{(x_i-m(\varepsilon))^2}{2\sigma(\varepsilon)^2}]$$

RESULTS

Apart from the above fuzzy classification we perform a blind bayesian classification to the two class random field using the SEM algorithm and the maximization of the posterior likelihood .We compare the estimated parameters and the error rates, which are computed as follows:

$$\tau = \frac{1}{card(S)}\sum_{s\in S}\left|\xi_s - \hat{\xi}_s\right|$$

Simulated image (fig. a)

b=1 d=0,8 g=0,95 a=1,4
20 iterations of the Gibbs sampler
200 iterations for the simulation of the fuzzy class

White noise

(1) $m_1=1$ $m_2=3$ $\sigma_1^2=\sigma_2^2=1$

	"Fuzzy SEM"	"Binary SEM"
m_1	0,97	0,93
m_2	3,03	2,72
σ_1	1,00	0,93
σ_2	0,97	1,08
π_1^2	0,35	
π_2^2	0,33	
τ	21,52	23,5

(2) $m_1=m_2=1$ $\sigma_1^2=4$ $\sigma_2^2=1$

	"Fuzzy SEM"	"Binary SEM"
m_1	0,98	0,98
m_2	1,00	1,00
σ_1^2	4,19	4,1
σ_2^2	0,99	1,3
π_1	0,30	
π_2	0,33	
τ	36,5	38,73

Correlated noise

(3) 8 neighbors $m_1=1$ $m_2=3$ $\sigma_1^2=\sigma_2^2=1$ (fig. b)

	"Fuzzy SEM"	"Binary SEM"
m_1	0,93	0,89
m_2	2,94	2,55
σ_1^2	1,00	1,05
σ_2^2	1,05	1,09
π_1	0,31	
π_2	0,33	
τ	21,43	25,6
	(fig. c1)	(fig. c2)

(4) 24 neighbours $m_1=1$ $m_2=3$ $\sigma_1^2=1$ $\sigma_2^2=4$

	"Fuzzy SEM"	"Binary SEM"
m_1	0,95	1,06
m_2	3,06	2,85
σ_1^2	0,96	1,09
σ_2^2	4,03	3,75
π_1	0,32	
π_2	0,31	
τ	26	27,4

In each case the fuzzy classification improves the error rates. Comparing case (1) and the case (2) the fuzzy classification seems to be less sensitive to spatial correlation. But, in the case of different deviations (cases (2) and (4)) its contribution is less significant. Particularly in the cases of correlated noises ((3) and (4)) the estimation of the parameters by the "fuzzy" SEM algorithm is more reliable.

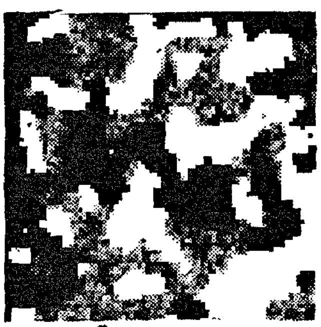
fig. a

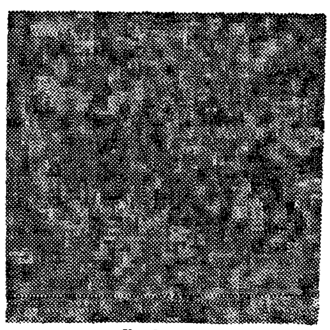
fig.b

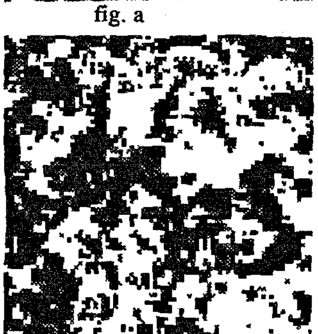
fig. c1

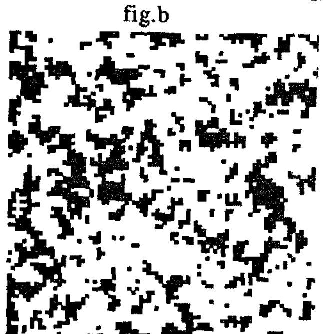
fig. c2

CONCLUSION

Numerous simulations show that our modelling includes many possibilities which can appear in reality. It remains valid for more than two pure classes and seems to be an alternative to Kent and Mardia's modelling ([4]) with the following important difference: for each pixel the prior probability distribution is concentrated in the set of configurations where no more than two classes are present. The model proposed in ([4]) assumes that this distribution is concentrated on the set of configurations where all classes are simultaneously present.

Insofar as our "fuzzy classification" is completely unsupervised and blind, the results are encouraging and could be certainly improved using contextual classification.

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