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Symbol Estimation with Channel State Information

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Outline

- Problem presentation
- Decoding algorithm by relaxation.
- Decoding algorithm by evaluation
- Simulations results
- Conclusion
Many communication problems can be modelized as:

\[
Y = H \cdot S + B
\]

Optimal decoding (ML):

\[
\hat{S} = \arg \min_{S \in \xi^n} \|Y - H \cdot S\|_2^2
\]
**Example 1: MIMO channel**

- Hypothesis: Rayleigh channel
  - *Quasi static channel, no frequency selectivity, perfect CSI*

- Modelization of the MIMO channel ($M$ transmitters, $N$ receivers)

\[
Y(k) = H_C X(k) + B(k)
\]

\[
\begin{bmatrix}
  y_1(k) \\
  \vdots \\
  y_N(k)
\end{bmatrix} =
\begin{bmatrix}
  h_{11} & \cdots & h_{MI} \\
  \vdots & \ddots & \vdots \\
  h_{1N} & \cdots & h_{MN}
\end{bmatrix}
\begin{bmatrix}
  x_1(k) \\
  \vdots \\
  x_M(k)
\end{bmatrix} +
\begin{bmatrix}
  b_1(k) \\
  \vdots \\
  b_N(k)
\end{bmatrix}
\]
Example 2: Space time coding

- Data information: $S \in \xi^{N_x}$
- Space time encoding: $S \rightarrow X = (x_1, x_2, ..., x_L)$
- with $X$ an $(N_t, L)$ matrix and $x_i$ is the $(N_t, 1)$ vector transmitted at time $i$ on the $N_t$ antennas.
- Received $y_i = H_c \times x_i + b_i \quad i = 1..L$

Since $X = M_cS$

$$Y = H_{ST}M_C S + B$$

$$Y = H_{ST}X + B$$
Example 3: MC-CDMA

- Based on a serial concatenation of direct sequence spreading with Orthogonal Frequency Division Multiplexing (OFDM)

- \( L \): number of carriers, users and size of spreading sequences

\[ y_1 = \sum_{i=1}^{L} h_1 \cdot c_{1,i} \cdot s_i \]

\[ y_2 = \sum_{i=1}^{L} h_2 \cdot c_{2,i} \cdot s_i \]

\[ y_L = \sum_{i=1}^{L} h_L \cdot c_{L,i} \cdot s_i \]

**Problem of type**: \( Y = H \cdot X + B \)
If equation given with complex number:

\[ Y_C = H_C \cdot S_C + B_C \]

Go back to real number using:

\[ Y_R = H_R S_R + B_R = \begin{bmatrix} re(Y_C) \\ \text{im}(Y_C) \end{bmatrix} = \begin{bmatrix} re(H_C) & -\text{im}(H_C) \\ \text{im}(H_C) & re(H_C) \end{bmatrix} \begin{bmatrix} re(S_C) \\ \text{im}(S_C) \end{bmatrix} + \begin{bmatrix} re(B_C) \\ \text{im}(B_C) \end{bmatrix} \]

\[ H \]
Decoding techniques

Two types of decoding algorithms

- **Decoding by relaxation**: use optimisation method on a convex set $V$ that includes $\xi^n$. Then, project the solution on the set $\xi^n$.
  - Zero Forcing,
  - MMSE
  - PIC, SIC
  - SPD

- **Decoding by evaluation**: select a subset of point $E$ of $\xi^n$. Then, evaluate:

$$\hat{S} = \arg\min_{S \in E} \|Y - H \cdot S\|_2^2$$

- Maximum Likelihood ($E = \xi^n$).
- Sphere Decoding: the set $E$ is constructed dynamically
- HISD (new algorithm)
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**Decorrelator (Zero Forcing):** $V = \mathbb{R}^n$

\[ S_r = \arg\min_{S \in \mathbb{R}^n} \| Y - H \cdot S \|_2^2 = H_{inv} \cdot Y \]

- $H_{inv} = (H^H H)^{-1} H^H$
- Projection onto $\xi^n$: $\tilde{S} = P_{\xi^n}(S_r)$

**Complexity:**
- Matrix inversion: $O(n^3)$
- For every points: $O(n^2)$
Generalised MMSE : $V = \text{sphere of diameter } n$

$$P_s : \arg\min_S \|Y - H \cdot S\|_2^2$$

$$\|S\|_2^2 \leq n$$

$$P_{eq} : \begin{cases} \max \min_{S, \lambda} \Gamma(S, \lambda) \\ \Gamma(S, \lambda) = \|Y - H \cdot S\|_2^2 + \lambda\left(\|S\|_2^2 - n\right) \end{cases}$$

$$S(\lambda) = \left(H^T H + \lambda I\right)^{-1} H^T Y$$

Projection onto $\xi^n : \hat{S} = P_{\xi^n} (S(\lambda))$

Complexity:
- Matrix inversion: $O(n^3)$
- Per point: $O(n^2)$

Note: if $\lambda = \sigma^2$ => MMSE decoder
Détecteurs SIC et PIC : $V = [-1,1]^n$

$\arg\min_{S \in [-1,1]^n} f(S) = \|Y - H \cdot S\|$

- Idea: projected gradient

- $O(Kn^2)$ Complexity, with $K$ the number of iterations.

- Projection onto $\bar{\xi}^n : \bar{S} = P_{\bar{\xi}^n}(S^{(K)})$

10x10 matrix $H$
SDP Detector: $V = [-1,1]^{n+1}$

- Idea: add a dummy dimension
  \[
  \arg\min_{S \in [-1,1]^n} \|Y_R - H_R S\|^2_2
  \]
  \[
  = \arg\min_{S' \in [-1,1]^{n+1}, S'(1) = 1} S'^t \begin{bmatrix} Y^t Y & -Y^t H \\ -H^t Y & H^t H \end{bmatrix} S'
  \]

- Non trivial iterative process
- Projection onto $\xi^{n+1}, \tilde{S} = P_{\xi^{n+1}}(S''(K))$
- Solution in $\xi^n$:
  \[
  \begin{cases}
  \text{if } S' = (1, s) & \tilde{S} = s \\
  \text{if } S' = (-1, s) & \tilde{S} = -s
  \end{cases}
  \]

- $O(K n^2)$ Complexity, with $K$ number of iterations

10x10 matrix $H$
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**Maximum Likelihood**

The optimal solution is:

\[ \hat{S} = \arg\min_{s \in \xi^n} \| Y - H \cdot S \|_2^2 \]

- \( \text{Card}(\xi^n) = 2^n \Rightarrow \text{exponential complexity in } n \)
- Evaluation of a point: \( O(n^2) \)

\[ \Rightarrow \text{Total Computational cost: } O(n^2 2^n) \]

- Good properties for VLSI implementation

- In practice, limited to small values of \( n \)
**Branch and Bound algorithm (1): let $S(1)=-1$**

$$\|Y - H \cdot S\|^2_2 = \text{Cte} / S(1) = -1$$

$$\tilde{S}^- = P_{\xi^n}(S_{r-})$$

$$\text{Min}^- = \|Y - H \cdot S_{r-}\|^2_2$$

$$\text{Max}^- = \|Y - H \cdot \tilde{S}_{r-}\|^2_2$$

*Plan $x_1=-1$ of $R^3$*

The optimal solution $\hat{S}_{r-}$ among the point $S(1)=-1$ verifies

$$\text{Min}^- \leq \|Y - H \cdot \hat{S}_{r-}\|^2_2 \leq \text{Max}^-$$
**Branch and Bound algorithm (2)**

→ Find optimal solution in a tree:

\[ Y \]

\[ YZ \]

\[ ZF \] decoding

\[ \Rightarrow \]

\[ \begin{align*}
\text{Find the point } S_{r-} \text{ of } -1 \times \mathbb{R}^{n-1} \text{ that minimizes} \\
\text{Min}^- &= \left\| Y - H \cdot S_{r-} \right\|_2^2 \\
\text{Max}^+ &= \left\| Y - H \cdot \tilde{S}_{r-} \right\|_2^2
\end{align*} \]

\[ \Rightarrow \]

\[ \text{Project } S_{r-} \text{ on } \xi^n : \tilde{S}^- = P_{\xi^n}(S_{r-}) \]

\[ \text{Let } \hat{S}_{r-} \text{ be the optimal solution of this branch} \]

\[ Min^- \leq \left\| Y - H \cdot \hat{S}_{r-} \right\|_2^2 \leq Max^- \]
→ Find optimal solution in a tree:

\[ Y \]

\[ \begin{align*}
(-1, y_2, \ldots, y_n) & \quad (1, y_2, \ldots, y_n) \\
\text{ZF} & \quad \text{ZF} \\
\text{Min}^-, \text{Max}^- & \quad \text{Min}^+, \text{Max}^+ \\
\end{align*} \]

\[ \begin{align*}
(-1, -1, y_3, \ldots, y_n) & \quad (-1, +1, \ldots, y_n) \\
\text{ZF} & \quad \text{ZF} \\
\text{Min}^-, \text{Max}^- & \quad \text{Min}^+, \text{Max}^+ \\
\end{align*} \]

if \( \text{Min}^- > \text{Max}^+ \) then the optimal solution is not in this branch => suppress it

Once there is no more branch to explore, the algorithm ends
Many variations on the same principle
- Ordering of the variables
- Exploration strategy of the tree.
- Computation of $S_r$.

Note: Sphere decoding is a particular realization of the BB algorithm

Average complexity is $O(n^3)$ BUT:
- sequential algorithm
- number of branches to explore is context dependant
  => not adapted for hardware implementation

Use sub-optimal algorithm (example: limit the number of branches to be explored).
HISD

→ New method proposed by the authors with two parameters: \((D, M)\):

→ General structure:
  - Find \(S_r\) using ZF.
  - From \(S_r\) and \(H\), select a list of potential candidates \(E\).
  - Evaluate all points of \(E\). If needed, increase the size of \(E\).
    - On average, \(4n(D + M)\) points evaluated
  - Parallel algorithm
  - Only 2 iterations on average
  - Well suited for hardware implementation
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Comparison of the algorithms

10x10 matrix $H$
Comparison of the algorithms

50x50 matrix $H$
HIS Performance with MC-CDMA

HIS (D=4, M=4) MC-CDMA, 4-QAM, L=16
Performance of HIS en MIMO (ASTC)

\[ Y = H_{ST} M_{ASTC} S + B \]

\[ M_{ASTC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \varphi & 0 & 0 \\ 0 & 0 & \theta & -\theta \varphi \\ 0 & 0 & \theta & \theta \varphi \\ 1 & -\varphi & 0 & 0 \end{bmatrix} \]

\[
\begin{cases}
\{s_1, s_2, s_3, s_4\} \in QAM - 4 \\
\varphi = e^{j\lambda} \quad \lambda = 0.5 \quad \theta = \sqrt{\varphi}
\end{cases}
\]

MIMO (ASTC) TX=2, RX=2 & 4QAM
(BELFIORE ENST PARIS)
Performance of HIS en MIMO (ASTC)

\[ Y = H_{ST} M_{ASTC} S + B \]

\[ M_{ASTC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \phi & 0 & 0 \\ 0 & 0 & \theta & -\theta \phi \\ 0 & 0 & \theta & \theta \phi \\ 1 & -\phi & 0 & 0 \end{bmatrix} \]

\[ \begin{cases} s_1, s_2, s_3, s_4 \in QAM - 4 \\ \phi = e^{j\lambda} \quad \lambda = 0.5 \quad \theta = \sqrt{\phi} \end{cases} \]

MIMO (ASTC) TX=2, RX=3 & 4QAM
(BELFIORE ENST PARIS)
Performance of HIS en MIMO (ASTC)

\[ Y = H_{ST} M_{ASTC} S + B \]

\[
M_{ASTC} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & \phi & 0 & 0 \\
0 & 0 & \theta & -\theta \phi \\
0 & 0 & \theta & \theta \phi \\
1 & -\phi & 0 & 0
\end{bmatrix}
\]

\[
\{s_1, s_2, s_3, s_4\} \in QAM - 4
\]

\[
\phi = e^{i\lambda}, \quad \lambda = 0.5, \quad \theta = \sqrt{\phi}
\]

MIMO (ASTC) TX=2, RX=4 & 4QAM
(BELFIORE ENST PARIS)
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Conclusion

- Knowledge GAP: find efficient solution to that problem
- Many applications (MIMO, BICM, MC-CDMA)
- Work to do
  - generalized with MAQ-M, PSK 16, 64, 256.
  - Soft output decoding
  - Finite precision algorithm
- Physical realisation ...