Iterative Multiuser Decoding

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Motivation

optimal

very high complexity

low complexity

inefficient ???
Multiuser Efficiency

Multiuser efficiency measures uncoded error probability.

$\eta \in [0; 1]$

$(\pm \infty \text{ dB}; 0 \text{ dB})$
Loss due to Separation of Detection & Decoding

Complex AWGN channel, random spreading, many users

\[ \Delta I = \eta - 1 - \log \eta \quad [\text{nats}] \]


Loss the greater the worse detector performs.
Iterative Receiver

- Knowledge gained by the code laws improves detection
- Feedback of conditional probabilities
- Convergence not ensured
Dynamics of the Iterative Receiver

Asking about the convergence of the iterative receiver is equivalent to asking for the dynamics of a *multi-dimensional* non-linear system with a random excitation.
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Approximations:

1. The spreading sequences are random and statistically independent.

2. The number of users $K$ and the spreading factor $N$ are large, i.e. $K \gg 1 \ll N$. 
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**Approximations:**

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**System becomes one-dimensional.**
Optimal Large System Multiuser Efficiency

\[
\frac{1}{\eta_l} = 1 + \frac{K}{N} \int \gamma \left(1 - p^2\right) \int \frac{1 - \tanh \left(z \sqrt{\gamma \eta_l} + \gamma \eta_l\right)}{1 - p^2 \tanh^2 \left(z \sqrt{\gamma \eta_l} + \gamma \eta_l\right)} Dz \ dF(\gamma, p)
\]

with

\[
p_{k,n} = 2 \Pr \left(x_{k,n} = 1 \mid \text{iteration } \ell - 1\right) - 1,
\]

the signal-to-noise ratio \(\gamma_k\) and the Gaussian measure \(Dz \triangleq \frac{\exp(-z^2/2)}{\sqrt{2\pi}} dz\).

(cond.) **LMMSE Large System Multiuser Efficiency**

Performance of best linear MMSE detector

\[
\frac{1}{\eta_\ell} = 1 + \frac{K}{N} \int \frac{\gamma (1 - p^2)}{1 + \eta_\ell \gamma (1 - p^2)} dF(\gamma, p)
\]

The interference powers in the filter reflect the dynamics of the interference power level from iteration to iteration.

(uncond.) LMMSE Large System Multiuser Efficiency

Performance of suboptimal linear MMSE detector

\[
\frac{1}{\eta_\ell} = 1 + \frac{K}{N} \int \frac{\gamma \int 1 - p^2 dF(p|\gamma)}{1 + \eta_\ell \gamma \int 1 - p^2 dF(p|\gamma)} dF(\gamma)
\]

The interference powers in the filter reflect the dynamics of the average interference power level from iteration to iteration.

Gaussian Approximation for the Decoder Output

The distribution $F(p|\gamma)$ of the decoder output signal can be well approximated by a single parameter $\mu(\gamma\eta)$ which characterizes the average reliability of the decoder output. [Gaussian approximation for the conditional log-likelihood ratios].

This average reliability depends on the structure of the codes, the signal-to-noise ratio $\gamma$ and the multiuser efficiency $\eta$.

The scalar function

$$\mu(\gamma\eta)$$

can be determined by simulation and interpolation and fully characterizes the code properties (within the limits of the Gaussian approximation).
Dynamics of the Iterative Receiver

\[ \eta \leftrightarrow \mu \leftrightarrow F(p|\gamma) \leftrightarrow \eta_{\ell+1} \]

The multiuser efficiency characterises the dynamics of the iterative receiver.
Convergence Properties

\[ K = \frac{3}{2} N \]
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\[ K = 3N \]
Power Optimisation

Convergence can be improved by disuniformising the users’ power levels.
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In the large system limit the power distribution with the minimum average power that allows for convergence of the multiuser efficiency towards at target multiuser efficiency (normally \( \approx 1 \)) can be found by solving a linear program.

Example of Optimal Power Profile

64 states convolutional code

BER = $10^{-5}$
Rate = $1/2$
Evolution Characteristic

\[ K = 3N \]
![Convolutional Code Graph]

**Convolutional Code**

- **Upper Bound**
- **Equal Power**
- **Without Iterations**

**Spectral Efficiency [bit/s/Hz]** vs. **Signal-to-noise Ratio [dB]**
Misconceptions Debunked

“If the load is too high, iterations do not converge.”
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Appropriate power assignment ensures convergence for any load.
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“If the load is too high, iterations do not converge.”
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“You need weak codes for iterative multiuser decoding to converge.”
Appropriate power assignment ensures convergence for any code. The stronger the code the higher is power efficiency.
Lessons Learned

Misconceptions Debunked

“If the load is too high, iterations do not converge.”
Appropriate power assignment ensures convergence for any load.

“You need weak codes for iterative multiuser decoding to converge.”
Appropriate power assignment ensures convergence for any code. The stronger the code the higher is power efficiency.

“Apply sphere decoding to improve on the linear MMSE detector.”
Misconceptions Debunked

“If the load is too high, iterations do not converge.”
appropriate power assignment ensures convergence for any load.

“You need weak codes for iterative multiuser decoding to converge.”
appropriate power assignment ensures convergence for any code.
The stronger the code the higher is power efficiency.

“Apply sphere decoding to improve on the linear MMSE detector.”
There is little gain over the linear MMSE detector.
The stronger the code the less gain is possible.
Open Problems
Open Problems

Go cellular
Open Problems

Go cellular

Include channel estimation
Open Problems

Go cellular

Include channel estimation

MIMO channels?
Open Problems

Go cellular

Include channel estimation

Correlated MIMO channels
Where Got the EXIT Charts Lost?

Instead of tracing mutual informations, multiuser efficiency was tracked.
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For any AWGN channel, any channel input distribution, the non-linear MMSE satisfies

\[ \text{MMSE} \left( \text{SNR} \right) = 2 \frac{\partial}{\partial \text{SNR}} I \left( \text{SNR} \right). \]