Superposition Coding for Costa Channels

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Outline

- Costa (Dirty Paper) Channels.
- Superposition Coding.
- Random-Coding Analysis.
- Simulation Results.
Costa Channels: Gaussian Dirty Paper

Channel Model:

\[ Y = X + S + Z \]

- \( P_X \), power constraint.
- \( Z \sim \mathcal{N}(0, P_Z) \), unknown noise.
- \( S \), interference known at encoder only.
Capacity:

Costa, 1983: Although $S$ known only at encoder,

\[ C = \frac{1}{2} \log \left( 1 + \frac{P_X}{P_Z} \right). \]

A special case of Gel’fand-Pinsker ’80

\[ C = I(U; Y) - I(U; S) \]

\[ U = X + aS \]

\[ a = \frac{P_X}{P_X + P_Z}. \]
Applications:

- Digital Watermarking.
- Broadcast Channel (MIMO).
- Interference Cancellation
  (also: broadcast with known interference).
- ISI Channels.
Question:

How to construct effective codes that approach capacity?

Answer 1: Nested lattices (Zamir, Shamai, Erez)

Answer 2: Superposition coding
Superposition Coding

Two Codes:

- $C_0$ quantization code.
- $C_1$ auxiliary code.

Superposition Code:

$$C = C_0 + C_1$$

$$= \{ c_0 + c_1 : c_0 \in C_0, c_1 \in C_1 \}$$
$C_1$
\[ C_0 + C_1 = \{ c_0 + c_1 : c_0 \in C_0, \ c_1 \in C_1 \} \]
Concepts of Encoding/Decoding:

Note:

- Discussion now is not rigorous!

Rigorous development:

Encoding:

Begins by selecting $c_1 \in C_1$
The bin associated with $c_1$ is

$$c_1 + C_0 = \{c_1 + c_0 : c_0 \in C_0\}$$
Interference:
s + x + z
\[ s + x + z = y \]
The Decoder:

\[ y = c_0 + c_1 + z \]
The Encoder:

- Size of $C_1$ determines number of messages.
- Larger $C_1$ $\implies$ higher transmission rate.
The bins:

\[ \text{bin} = c_1 + C_0 = \{c_1 + c_0 : c_0 \in C_0\} \]

- Too small \( \implies x \) violates power constraint.
Rigorous Development:

- Modulo-$A$ arithmetic reduces signals to a cube $[-A/2, A/2]^N$.
- A *dither* $\mathbf{d}$, uniformly distributed in $[-A/2, A/2]^N$.
- $\alpha$-scaling (MMSE) is applied.
Random Coding Analysis

- $C_0 \sim \text{Unif} \left(-A/2, A/2\right)$, rate $R_0$.
- $C_1 \sim \mathcal{N}(0, Q)$, rate $R_1$. 
Theorem 1.

Decoding possible if \((R_0, R_1)\) in MAC capacity region.

Power constraint obeyed if \(R_0\) above dashed line.
Achievable Region

\[ R_1 = \frac{1}{2} \log \left( 1 + \frac{P_X}{P_Z} \right) \] achieved!
\[ R_1 = \frac{1}{2} \log \left( 1 + \frac{P_X}{P_Z} \right) \]

\[ Q \geq \frac{P_X^2}{P_X + P_Z} \implies \text{Capacity achieved.} \]
\[ R_1 = \frac{1}{2} \log \left( 1 + \frac{P_X}{P_Z} \right) \]

Achievable Region

\[ Q = \frac{P_X^2}{P_X + P_Z} \implies \text{Capacity achieved at vertex point.} \]
Comparison

- Superposition coding:

  Prefers:

  \[ Q = \frac{P_X^2}{P_X + P_Z} < P_X \]

- Nested lattices:

  Similar to

  \[ Q = P_X \]
Simulation Results

**Codes:**

- $C_0$ Ungerboeck trellis code.
- $C_1$ LDPC code.

**MAC joint iterative decoder**

- $C_0$ by BCJR.
- $C_1$ by belief propagation.
Simulation results:

Rate = 0.25 bits/channel use

Gap to Shannon limit = 1.3 dB

Discussion:

Why 0.25? : Dirty tape (Shannon’s 58 causal case): 3.2 dB.

Comparison, Nested lattices (Erez, ten Brink): 1.3 dB.

Recent work: Optimized Source (TCQ) & Channel (IRA) Coding & improved joint detection (Sun-Liveris-Stankovic-Xiong, CISS 2005): 1. dB.
Comparison, Nested Lattices

Different geometry of codes:

\[ Q = \frac{P_X^2}{P_X + P_Z} < P_X. \]

Simplicity of superposition coding:

- Conceptual and structural simplicity.
- Encoder (no finding coset leader).
- MAC Decoder.
Prospects

- Generalization to arbitrary side-information (Gel'fand Pinsker) problem.
- Improving on code selection ($C_0$-quantization code).
- Vector Channels:

\[ y = Hx + s + n. \]

- Direct extension of superposition coding to vector channel:
  (S.-C. Lin and H.-J. Su, CISS 2005)

\[ x = (c_0 + c_1 - Ws - U) \mod A \]

$W$-scaling MMSE matrix.
- Our SVD approach $H = \psi \Lambda \phi \Rightarrow$ equivalent parallel scaled scalar channels with modified but known interference:

\[ y_i = \lambda_i x'_i + s'_i + n_i, \quad i = 1, 2 \ldots \text{rank } (H). \]
Backup Slides
**Choice of $C_0$:**

- bin = Translation of $C_0$.
- Finding closest word to $s$ requires: $C_0$ be good quantization code.
- Quantization with LDPC: open problem.
$C_1$ uniformly in Voronoi of $C_0$.

Equivalent to $Q = P_X$. 
Superposition Coding

\[ Q = \frac{P_X^2}{P_X + P_Z} < P_X. \]

\[ \text{Prefers} \]