Performance of large communication systems: recent works and projects.

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Existing collaborations.

Inside NEWCOM.

- Walid Hachem (Supelec)
- Merouane Debbah (Eurecom)

Outside NEWCOM.

- With mathematicians in the context of a French research project.
  - J. Najim (Telecom Paris, France)
  - O. Khorunzhy (Université de Versailles, France)
  - L. Pastur (Institute of low temperature physics, Ukraine)
- V. Girko (6 months stay at Marne la Vallée beginning in April 2005)
Recent works I.

Large downlink CDMA systems.

- Performance of (reduced-rank) Wiener in the context random Haar distributed orthogonal codes
- Performance of (reduced-rank) chip-rate Wiener equalizer followed by despreading in the context of scrambled Walsh-Hadamard matrices
Recent works II.

Mutual information of correlated Ricean MIMO channels.

\[ \tilde{\mathbf{H}} = \tilde{\mathbf{A}} + \tilde{\mathbf{Y}} \]

- \( \tilde{\mathbf{A}} \) possibly full rank
- \( \tilde{\mathbf{Y}} \) Gaussian centered, \( E(\tilde{\mathbf{Y}}_{m,n} \tilde{\mathbf{Y}}^*_{p,q}) = R(m-p, n-q) \)

\[
C(\sigma^2) = \int_{\sigma^2}^{+\infty} \left( \frac{1}{\omega^2} - \frac{1}{r} \text{Trace}(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H + \omega^2 \mathbf{I})^{-1} \right) d\omega^2 \\
\approx \int_{\sigma^2}^{+\infty} \left( \frac{1}{\omega^2} - m_r(\omega^2) \right) d\omega^2
\]

for some deterministic function \( m_r \).

Provides a relevant and useful approximation of \( E(C(\sigma^2)) \)
Recent works III.

Equivalent to study.

\[ H = A + Y \]

- \( A \) possibly full rank
- \( Y \) Gaussian, zero mean, independent entries with different variances

Existence of a deterministic function \( m_r(\sigma^2) \) such that

\[
\frac{1}{r} \text{Trace} \left( \left( HH^H + \sigma^2 I \right)^{-1} \right) \simeq m_r(\sigma^2)
\]

Relevant and useful approximation of \( \mathbb{E}(C(\sigma^2)) \) by

\[
\overline{C}(\sigma^2) = \int_{\sigma^2}^{+\infty} \left( \frac{1}{\omega^2} - m_r(\omega^2) \right) d\omega^2
\]
Recent works IV.

Based on results of Girko.

Obtained under the non satisfying assumption that the $\| \cdot \|_1$ norms of the rows and of the columns of $\mathbf{A}$ are bounded.

Mathematical work to extend the result to the boundness of the $\| \cdot \|_2$ norms.

Extension of the approach used recently by Dozier-Silverstein.
Projects.

Large CDMA systems.

• Performance of MMSE receivers when the intersymbol interference is not negligible.

\[ y(n) = H_0 W(n)s(n) + H_1 W(n - 1)s(n - 1) + v(n) \]

Study of matrix \( H_0 W(n)W(n)^H H_0^H + H_1 W(n - 1)W(n - 1)^H H_1^H \) with i.i.d. or Haar distributed codes.

Time-varying code (\( W(n) \) and \( W(n - 1) \) independent) or time-invariant code (\( W(n) = W(n - 1) \)), estimation of \( s_1(n) \) based on \( y(n) \), or \( y(n), y(n - 1), ... \).

• Performance of MMSE receivers in the context of scrambled Walsh-Hadamard codes.

\[ W(n) = \text{Diag}(s_1(n), \ldots, s_N(n))C \]
Projects.

MIMO systems.

- Gaussian approximation of the mutual information of correlated Ricean MIMO channels.
  
  No existing mathematical results.
Projects.

G-estimation and applications.

Introduced by Girko.

Application of large random matrix theory to statistical estimation.

Example 1: estimation of the SINR in a Downlink CDMA system.

\[ y(m) = HWs(m) + v(m) \]

Estimation of the SINR of the user 1 MMSE receiver \( \frac{\eta}{1-\eta} \), \( \eta = w_1^H H^H R^{-1} H w_1 \).

\( w_1 \) and \( H \) are known, but \( R \) is unknown. Standard solution:

\[ \hat{R} = \frac{1}{M} \sum_{m=1}^{M} y(m)y(m)^H, \hat{\eta} = w_1^H H \hat{R}^{-1} H^H w_1 \]

Not justified because if \( M \approx N \), \( \hat{R} \) is not a consistent estimator of \( R \).
Projects.

$G$ – estimation and applications.

**Example 2 : estimation of the mutual information.**

$\mathbf{H}$ MIMO channel, $C(\sigma^2) = \frac{1}{r} \log \det (\mathbf{I} + \frac{\mathbf{H}\mathbf{H}^H}{\sigma^2})$

$\hat{\mathbf{H}} = \mathbf{H} + \mathbf{N}$ measured channel, $\mathbf{N}$ Gaussian i.i.d. matrix.

$\frac{1}{r} \log \det (\mathbf{I} + \frac{\hat{\mathbf{H}}\hat{\mathbf{H}}^H}{\sigma^2})$ poor estimate of $C(\sigma^2)$.

If $r$ and $t$ large enough, possible to estimate consistently $C(\sigma^2)$ from $\hat{\mathbf{H}}$. 